

Sum-of-Norms Model Predictive Control for Spacecraft Maneuvering

Mirko Leomanni, Gianni Bianchini, Andrea Garulli, Antonio Giannitrapani, Renato Quartullo

Abstract—This paper tackles spacecraft optimal control problems in which the cost function is defined by a sum of vector norms, in order to optimize fuel consumption while achieving sparse actuation. An MPC strategy is devised for such type of problems, accounting for different spacecraft maneuvering modes. Closed-loop stability is guaranteed by a conic Lyapunov function, which is employed as a terminal cost in the formulation. A systematic method to construct such function is presented. The proposed design is compared to a standard quadratic MPC scheme on a long-range rendez-vous mission.

I. INTRODUCTION

Model Predictive Control (MPC) has a tremendous potential to shape the future of aerospace control systems. This observation is supported by a number of factors including, first and foremost, the ability to systematically handle constraints and to optimize relevant performance indexes, see, e.g., [1] and references therein.

Spacecraft maneuvering problems are different from process control problems for which MPC was originally conceived, mainly because fuel consumption often represents the most important performance metric. Although this is an obvious notion in astrodynamics, most often the spacecraft maneuvering objective is still cast in terms of a quadratic cost function in the aerospace control literature. The drawbacks of this approach are suboptimal propellant utilization and undesirable continuous thrusting. In order to weigh fuel consumption, the cost function should instead be specified as a sum of vector p -norms. Values of $p = 2$ or $p = 1$ are of interest in aerospace applications [2]. When $p = 1$ and the dynamics are linear, the optimal control problem can be cast as a linear programming (LP) problem. LP-based MPC has been studied extensively, see, e.g., [3], [4], [5]. In these papers, input sparsity has been reported as a prominent control feature. Indeed, it has been recently shown in [6], [7] that, under certain assumptions on the system structure, the optimization of a linear cost gives the sparsest admissible control.

Vector norm regularized MPC has been also investigated thoroughly in the literature, with the aim of promoting control sparsity. In [8], [9], the standard quadratic cost is augmented with a sum of 1-norms, so as to decrease the number of active control input channels. A drawback of this method is the requirement to tune the regularization parameter empirically. In order to overcome this issue, ℓ_0 -constrained quadratic MPC has been proposed in [10], [11].

The authors are with the Dipartimento di Ingegneria dell'Informazione e Scienze Matematiche, Università di Siena, Siena, Italy. Email: {leomanni,giannibi,garulli,giannitrapani}@dii.unisi.it, renato.quartullo@gmail.com.

Unfortunately, this approach leads to a nonconvex optimization problem.

A fundamental issue in MPC design is how to guarantee closed-loop stability. A simple method is to constrain the terminal state to be exactly equal to zero [12]. However, this usually sacrifices performance and raises feasibility issues. A less conservative approach is to embed a stabilizing terminal state penalty in the MPC cost function [13]. Most of the stability results within this setting consider a quadratic cost. In the LP formulation, stability is achieved by adopting a suitable polyhedral Lyapunov function as a terminal cost. Polyhedral Lyapunov functions provide a powerful and flexible tool for stability analysis [14], [15]. However, with the exception of few specific results (see, e.g., [16]), they are difficult to compute. It is known, for instance, that their complexity may grow indefinitely when the system eigenvalues approach the unit circle [17].

In this paper, the sum-of-norms MPC problem is addressed for spacecraft translational maneuvering. A sum of 2-norms ($p = 2$) MPC scheme is developed for applications featuring thrust vector control. The proposed design guarantees exponential stability by means of a conic Lyapunov function, which is employed as a terminal cost in the formulation. A systematic method to construct this function, from the solution of a Lyapunov equation, is presented. A terminal constraint set is included in the formulation, thus avoiding the issues related to the presence of a terminal equality constraint. Such type of construction can be easily generalized to MPC problems in which the stage cost is specified via a sum of arbitrary p -norms, such that $p \geq 1$. The case $p = 1$ is discussed for spacecraft equipped with orthogonal thrusters. Contrary to LP-based MPC, the resulting control scheme does not require the computation of a polyhedral Lyapunov function. With respect to MPC schemes proposed in the aerospace literature (e.g., [8], [18], [19]), the main novelty is the adoption of a sum-of-norms cost function coupled to a stability proof which does not require a terminal equality constraint.

The MPC schemes obtained for $p = 2$ and $p = 1$ are demonstrated on a simulation case study of a rendez-vous mission. It is observed that both controllers achieve finite time convergence, as well as control sparsity. Moreover, the resulting fuel consumption is lower than that provided by a standard MPC scheme employing a quadratic cost.

The paper is organized as follows. The main features of the spacecraft maneuvering problem are described in Section II, and the proposed MPC design is presented in Section III. The rendez-vous case study is discussed in Section IV, and conclusions are drawn in Section V.

II. PROBLEM SETTING

The focus of this paper is on spacecraft translational maneuvering problems involving linear discrete-time dynamics of the form

$$x(k+1) = Ax(k) + Bu(k), \quad (1)$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$. More specifically, the state $x(k)$ is a six-dimensional vector ($n = 6$) describing the relative motion between a spacecraft and another orbiting body, and the dynamics (1) are obtained by linearizing the relative motion about the equilibrium configuration $x = 0$. Several different parameterizations of the relative motion have been proposed in the literature, using, for instance, relative orbital elements or relative position and velocity, see [20]. In this paper, the orbital-element parameterization in [21] is adopted. A key feature of such parameterization is that the resulting linearized model accurately describes the relative motion of two bodies in a circular orbit, even for an arbitrarily large angular separation.

The control input $u = [u_R, u_T, u_N]^T$ is a three-dimensional vector ($m = 3$) which represents the spacecraft propulsion system thrust vector, expressed in a Radial-Transversal-Normal (RTN) coordinate frame centered at the spacecraft. The R-axis lies along the radius vector joining the spacecraft and central body. The T-axis is tangential to the orbit and aligned with the spacecraft velocity vector. The N-axis is normal to the orbital plane and completes a right handed triad, as shown in Fig. 1. More details about the system model are given in Section IV.

The maneuvering objective consists of steering system (1) to the equilibrium point $x = 0$, while satisfying predefined control bounds. Since the amount of propellant carried by a spacecraft is severely constrained, fuel consumption is the primary performance metric. Moreover, it is often desired to keep the thruster usage at minimum, since thrusting can interfere with the functioning of onboard sensors. The following control modes can be adopted, depending on the mission design [2]:

- (A) **Thrust vectoring**: maneuvering is achieved by firing a single thruster and steering the thrust vector via attitude control. In this approach, $\|u(k)\|_2$ is the thruster command while the azimuth and elevation angles of the thrust vector (see Fig. 1) are reference commands fed to the attitude control system. Constraints on the maximum deliverable thrust can be expressed as $\|u(k)\|_2 \leq 1$. The thruster fuel consumption is proportional to $\sum_k \|u(k)\|_2$.
- (B) **Orthogonal thrusters**: maneuvering is achieved by using a set of six identical thrusters aligned to the RTN axes (two for each axis in opposite directions). In this setting, the control bounds can be modeled as $\|u(k)\|_\infty \leq 1$. The fuel consumption of the propulsion system is proportional to $\sum_k \|u(k)\|_1$, i.e., to the sum of the fuel consumption of each individual thruster (thrusters in opposite directions are never fired in pairs).

Notice that, in the considered framework, both the system state and the input variables are quantities normalized by

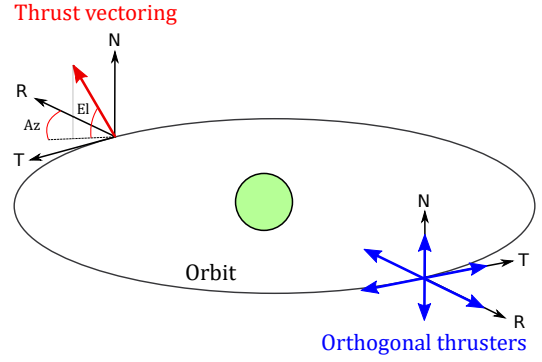


Fig. 1. Spacecraft control modes in the RTN frame.

the maximum control magnitude, which allows the input constraints to be expressed as in points (A)-(B). Moreover, the translational dynamics (1) are decoupled from rotational (attitude) dynamics. This is a reasonable assumption in practice, because the attitude control authority has typically a far higher bandwidth than the translational one [22].

The above control specifications naturally lead to the formulation of a constrained optimal control problem, in which the cost function is defined by a sum of norms. Such control problem is solved according to the receding horizon principle to yield a stabilizing MPC law, as discussed next.

III. MPC DESIGN

The optimal control problem addressed in this work is as follows

$$\min_{\hat{u}_k} \sum_{j=0}^{N-1} [\|Q\hat{x}_k(j)\|_p + \|\hat{u}_k(j)\|_p] + V(\hat{x}_k(N)) \quad (2)$$

$$\text{s.t. } \hat{x}_k(j+1) = A\hat{x}_k(j) + B\hat{u}_k(j) \quad (3)$$

$$\hat{x}_k(0) = x(k) \quad (4)$$

$$\|\hat{u}_k(j)\|_q \leq 1 \quad j = 0 \dots N-1 \quad (5)$$

$$\hat{x}_k(N) \in \mathcal{S} \quad (6)$$

where N is the prediction horizon, $\|\cdot\|_p$ denotes the vector p -norm, Q is a full rank weighting matrix, $V(\cdot)$ is a positive definite function, and $\mathcal{S}$ is a terminal set. The control sequence to be optimized is $\hat{u}_k = \{\hat{u}_k(0), \dots, \hat{u}_k(N-1)\}$. The cost function (2), besides the fuel consumption term $\sum_j \|\hat{u}_k(j)\|_p$, contains a term $\sum_j \|Q\hat{x}_k(j)\|_p$ weighting the state transient and a terminal cost $V(\hat{x}_k(N))$ which is instrumental for stability analysis. Matrix Q allows one to trade-off the relative influence of fuel consumption and state regulation. The two different control modes outlined in Section II are recovered by setting either $p = 2, q = 2$ (thrust vectoring) or $p = 1, q = \infty$ (orthogonal thrusters) in (2) and (5). The MPC strategy amounts to solving problem (2)-(6) at each discrete time step k and applying the control input

$$u(k) = \hat{u}_k(0) \quad (7)$$

to system (1).

The following result [13] is used to enforce closed-loop stability of the control system (1)-(7).

Lemma 1: Let system (1) be stabilizable under state feedback and K be a feedback gain, such that $A_{cl} = (A - BK)$ is Schur stable. Moreover, let the set $\mathcal{D}$ of initial conditions $x(0)$ for which problem (2)-(6) is feasible be nonempty. Assume that the two conditions below are satisfied:

(i) There exists $V(\cdot)$ such that the Lyapunov inequality

$$V(A_{cl}x) - V(x) \leq -\|Qx\|_p - \|Kx\|_p \quad (8)$$

is satisfied for all $x \in \mathcal{S}$.

(ii) $\mathcal{S}$ is a positively invariant set for system $x(k+1) = A_{cl}x(k)$, and such that $\|Kx\|_q \leq 1$ for all $x \in \mathcal{S}$.

Then, the equilibrium point $x = 0$ of the closed-loop system (1)-(7) is exponentially stable with domain of attraction $\mathcal{D}$.

Clearly, $V(\cdot)$ is a Lyapunov function for the closed-loop system $x(k+1) = A_{cl}x(k)$. Since the right hand side of (8) is upper and lower bounded by linear functions of $\|x\|_p$, Lemma 1 rules out the possibility of taking $V(\cdot)$ as a quadratic form (in which case, $V(A_{cl}x) - V(x)$ is sublinear near $x = 0$). Instead, $V(\cdot)$ can be defined in terms of a vector norm. In this paper, the weighted 2-norm

$$V(x) = \|Wx\|_2 \quad (9)$$

is preferred to a polyhedral function (e.g., of the form $\|Wx\|_1$ or $\|Wx\|_\infty$) for ease of computation. Notice that, for the choice (9) and the considered thrusting modes, (2)-(6) is a convex optimization problem that can be cast as a second order cone program (SOCP). In the following, it is shown how to satisfy the two conditions of Lemma 1 within this setting.

A. Thrust vectoring

In the thrust vectoring scenario, one has that $p = 2$ in equation (8). Then, (8)-(9) result in

$$\|WA_{cl}x\|_2 - \|Wx\|_2 \leq -\|Qx\|_2 - \|Kx\|_2. \quad (10)$$

Let the matrix P be the positive definite solution to the Lyapunov equation

$$A_{cl}^T P A_{cl} - P + C = 0, \quad (11)$$

where C is a symmetric positive definite matrix. In this paper, the terminal weight W in (10) is parameterized as follows

$$W = \alpha Y, \quad (12)$$

where $\alpha > 0$ is a scaling parameter, and Y is given by the Cholesky decomposition of $P = Y^T Y$.

The following results characterize two possible choices of the scaling parameter α , guaranteeing that (10) is satisfied for all $x \in \mathbb{R}^n$.

Proposition 1: Let

$$\alpha = \frac{(\|Q\|_2 + \|K\|_2)(\|YA_{cl}\|_2 + \|Y\|_2)}{\lambda_{\min}(C)} \quad (13)$$

where $\|\cdot\|_2$ denotes the induced matrix 2-norm and $\lambda_{\min}(\cdot)$ indicates the minimum eigenvalue of a matrix. Then, W defined by (11)-(13) satisfies (10).

Proof: Equation (11) implies

$$x^T (A_{cl}^T Y^T Y A_{cl}) x - x^T Y^T Y x = -x^T C x. \quad (14)$$

Equation (14), in turn, can be rewritten as

$$\|YA_{cl}x\|_2 - \|Yx\|_2 = \frac{-x^T C x}{\|YA_{cl}x\|_2 + \|Yx\|_2}. \quad (15)$$

Taking into account (12), and using a standard upper bound for the expression on the right hand side of (15), one gets

$$\begin{aligned} \|WA_{cl}x\|_2 - \|Wx\|_2 &\leq -\alpha \frac{\lambda_{\min}(C) \|x\|_2^2}{(\|YA_{cl}\|_2 + \|Y\|_2) \|x\|_2} \\ &= -\alpha \frac{\lambda_{\min}(C)}{\|YA_{cl}\|_2 + \|Y\|_2} \|x\|_2. \end{aligned} \quad (16)$$

Substituting (13) into (16) gives

$$\|WA_{cl}x\|_2 - \|Wx\|_2 \leq -(\|Q\|_2 + \|K\|_2) \|x\|_2, \quad (17)$$

which, being $(\|Q\|_2 + \|K\|_2) \|x\|_2 \geq \|Qx\|_2 + \|Kx\|_2$, indicates that (10) is satisfied. ■

An alternative characterization, which is inspired by the results in [4], [16], can be given as follows.

Proposition 2: Let

$$\alpha = \frac{\|QY^{-1}\|_2 + \|KY^{-1}\|_2}{1 - \|H\|_2} \quad (18)$$

where $H = YA_{cl}Y^{-1}$. Then, W defined by (11)-(12) and (18) satisfies (10).

Proof: By definition $YA_{cl} = HY$. Then, by (12), $WA_{cl} = HW$. Condition (10) can be rewritten as

$$\|HWx\|_2 - \|Wx\|_2 + \|QY^{-1}Yx\|_2 + \|KY^{-1}Yx\|_2 \leq 0. \quad (19)$$

An upper bound for the expression on the left hand side of (19) is given by

$$(\|H\|_2 - 1) \|Wx\|_2 + (\|QY^{-1}\|_2 + \|KY^{-1}\|_2) \|Yx\|_2. \quad (20)$$

Substituting (12) into (20) gives

$$\left[(\|H\|_2 - 1)\alpha + \|QY^{-1}\|_2 + \|KY^{-1}\|_2 \right] \|Yx\|_2. \quad (21)$$

The above expression is equal to zero when α is set as in (18). Being $P = Y^T Y$ a solution to (11), one has that $\|H\|_2 < 1$ (see [16]) and therefore $\alpha > 0$. Hence, it follows that (19) and, equivalently, (10) are satisfied. ■

It is not difficult to verify that, for scalar control systems ($n = m = 1$), Propositions 1 and 2 give the same α . This does no longer hold for multivariable systems, as it will be shown in Section IV.

Now, consider point (ii) of Lemma 1. Being $q = 2$, one has to find a positively invariant set $\mathcal{S}$ in which $\|Kx\|_2 \leq 1$. To this aim, let $Z^T Z$ be the positive definite solution to the Lyapunov equation

$$A_{cl}^T Z^T Z A_{cl} - Z^T Z + D = 0, \quad (22)$$

where D is a symmetric positive definite matrix. Then, $\mathcal{S}$ can be taken as the largest sublevel set of $\|Zx\|_2$ in which $\|Kx\|_2 \leq 1$, i.e. [23, Sec. 8.4.2]

$$\mathcal{S} = \left\{ x : \|Zx\|_2 \leq \frac{1}{\|KZ^{-1}\|_2} \right\}. \quad (23)$$

Notice that C in (11) and D in (22) can be chosen independently. In other words, the computation of W is decoupled from that of $\mathcal{S}$.

B. Orthogonal thrusters

In the case of orthogonal thrusters, one has that $p = 1$ in equation (8) of Lemma 1. Hence, (8)-(9) result in

$$\|WA_{cl}x\|_2 - \|Wx\|_2 \leq -\|Qx\|_1 - \|Kx\|_1. \quad (24)$$

Similarly to (12), the terminal weight is parameterized as

$$W = \beta Y, \quad (25)$$

where Y is obtained from the solution $P = Y^T Y$ to (11). The following result characterizes a possible choice of β , guaranteeing that (10) is satisfied for all $x \in \mathbb{R}^n$.

Proposition 3: Let

$$\beta = \sqrt{\gamma} \alpha \quad (26)$$

where $\gamma = \max(n, m)$ and α is specified by Proposition 1 or 2. Then, W defined by (25)-(26) satisfies (24).

Proof: According to Propositions 1 and 2, one has

$$\|\alpha Y A_{cl}x\|_2 - \|\alpha Y x\|_2 \leq -\|Qx\|_2 - \|Kx\|_2, \quad (27)$$

By using norm inequalities

$$\sqrt{\gamma}(\|Qx\|_2 + \|Kx\|_2) \geq \|Qx\|_1 + \|Kx\|_1, \quad (28)$$

and therefore

$$\begin{aligned} \|\sqrt{\gamma}\alpha Y A_{cl}x\|_2 - \|\sqrt{\gamma}\alpha Y x\|_2 &\leq \\ -\sqrt{\gamma}(\|Qx\|_2 + \|Kx\|_2) &\leq -\|Qx\|_1 - \|Kx\|_1, \end{aligned} \quad (29)$$

which, being $\sqrt{\gamma}\alpha Y = W$, concludes the proof. ■

Concerning point (ii) of Lemma 1 with $q = \infty$, one has to guarantee that $\|Kx\|_\infty \leq 1$ within a positively invariant terminal set $\mathcal{S}$. To this aim, $\mathcal{S}$ can be selected as [23, Sec. 8.4.2]

$$\mathcal{S} = \left\{ x : \|Zx\|_2 \leq \frac{1}{\max_i \|(KZ^{-1})_i\|_2} \right\}, \quad (30)$$

where $(\cdot)_i$ denotes the i -th row of a matrix and Z is given by (22).

The results presented in this section can be extended to MPC problems including state constraints by defining $\mathcal{S}$ as the largest sublevel set of $\|Zx\|_2$ in which both state and input constraints are met.

IV. APPLICATION TO SPACE RENDEZ-VOUS

The proposed MPC design is demonstrated on a circular rendez-vous mission, in which a spacecraft is required to intercept another orbiting body. A discretized linear time-invariant model, based on relative orbital elements, has been derived for this problem in [21] and is given by (1) with

$$A = \begin{bmatrix} 1 & t_s & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos(t_s) & -\sin(t_s) & 0 & 0 \\ 0 & 0 & \sin(t_s) & \cos(t_s) & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos(t_s) & -\sin(t_s) \\ 0 & 0 & 0 & 0 & \sin(t_s) & \cos(t_s) \end{bmatrix}$$

$$B = \begin{bmatrix} -2t_s & -\frac{3}{2}t_s^2 & 0 \\ 0 & -3t_s & 0 \\ \cos(t_s) - 1 & 2\sin(t_s) & 0 \\ \sin(t_s) & 2(1 - \cos(t_s)) & 0 \\ 0 & 0 & \frac{1}{2}\sin(t_s) \\ 0 & 0 & \frac{1}{2}(1 - \cos(t_s)) \end{bmatrix},$$

where t_s is the sampling interval. The dimensional unit of t_s is radians per sample, where 2π radians corresponds to a full orbital period. A value of $t_s = \pi/32$ is selected in this study. Notice that all the eigenvalues of A lie on the unit circle. According to the setting in Section II, the three columns of B model the influence of radial, tangential and normal thrust inputs on the system dynamics.

The first component of the state vector $x \in \mathbb{R}^6$ in (1) describes the relative phase angle between the two orbiting bodies, and is proportional to the along-track separation. The second component indicates the relative angular velocity, which corresponds to the radial separation. The third and the fourth components are defined in terms of the relative eccentricity vector, which is close to zero for circular orbits. The last two components are proportional to the inclination vector and hence to the cross-track separation. Such parameterization allows one to effectively deal with large along-track separations. All state variables are normalized by a maximum control acceleration of $8 \cdot 10^{-4}$ m/s². The (normalized) initial condition for the maneuver is given by $x(0) = [157, 0, 0, 0, 1, 0]^T$, which corresponds to an along-track separation of 110 km and a cross-track separation of 1.4 km. These are consistent with long-range rendez-vous applications.

The auxiliary feedback gain K in Lemma 1 is chosen as the LQR state feedback matrix $K = (B^T \Psi B + I)^{-1} B^T \Psi A$, where Ψ is the solution to the discrete-time ARE

$$\Psi = A^T \Psi A - A^T \Psi B (I + B^T \Psi B)^{-1} B^T \Psi A + Q^T Q. \quad (31)$$

Notice that Ψ also solves the Lyapunov equation $A_{cl}^T \Psi A_{cl} - \Psi + Q^T Q + K^T K = 0$. The terminal set $\mathcal{S}$ is chosen as in (22)-(23) with $D = Q^T Q + K^T K$, which corresponds to $\mathcal{S}$ being a sublevel set of $x^T \Psi x$. This leaves the control horizon N , as well as Q in (2) and C in (11) as the degrees of freedom available for control tuning. The choice of N is a compromise between computational complexity and optimality, while the state weighting matrix Q allows one to trade off fuel consumption with tracking performance.

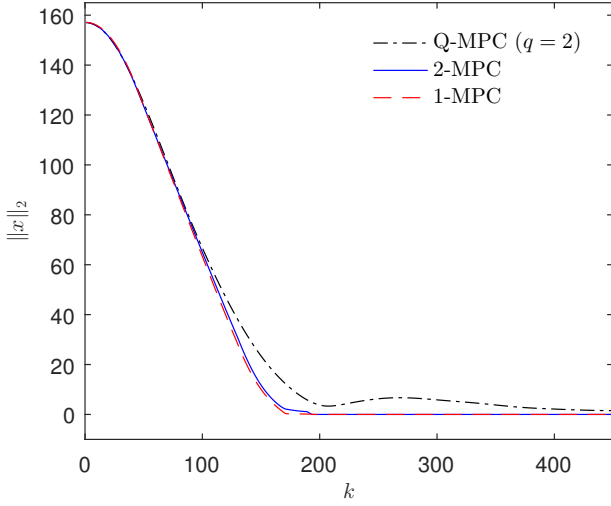


Fig. 2. 2-norm of the system state trajectories.

The tuning of the matrix C , which determines the stabilizing terminal weight W via Propositions 1-3, is more involved. Ideally, W should be made as small as possible (in norm) so as to favour fuel efficiency. To this aim, one can take the smaller of the two scalings α provided by Propositions 1 and 2. Nevertheless, W depends on C in a complex and nonintuitive manner. In particular, it should be noticed from (12)-(13) and (18) that scaling C by a scalar positive factor does not affect W . Hence, only a change in the structure of C may lead to a different result. To better illustrate this point, W is computed according to both Propositions 1 and 2, by setting

$$Q = 10^{-2} \cdot \text{diag}([1, 1, 1, 1, 2, 2]) \quad (32)$$

and using two different choices for the C matrix: $C = I$ and $C = Q^T Q + K^T K$. The obtained results are displayed in Table I. It can be seen that constructing the terminal cost from the solution to the ARE (31) does not necessarily result in a smaller W (see the third column of Table I). Moreover, either of the two propositions may provide a smaller α , depending on the choice of C . In what follows, it is assumed that $C = I$ and that α is specified via Proposition 1. A systematic method for exploiting the degrees of freedom provided by C for performance optimization is the subject of current investigation.

The rendez-vous maneuver is simulated numerically for $k = 450$ samples. The two MPC schemes detailed in Section III, hereafter denoted by 2-MPC (thrust vectoring mode) and 1-MPC (orthogonal thrusters mode), are compared to a standard MPC scheme based on a quadratic cost function, referred to as Q-MPC. The Q-MPC scheme is obtained by

TABLE I

TERMINAL WEIGHT MAGNITUDE

Type	$C = I$	$C = Q^T Q + K^T K$
Prop. 1	$\ W\ _2 = 157.1$	$\ W\ _2 = 1190$
Prop. 2	$\ W\ _2 = 249.3$	$\ W\ _2 = 692.8$

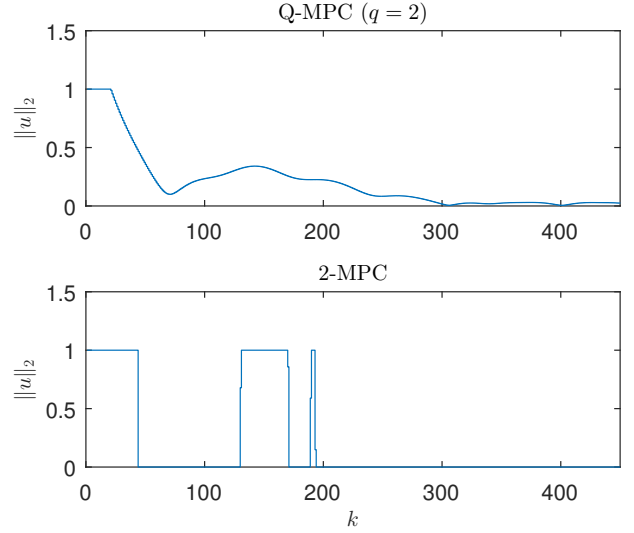


Fig. 3. 2-norm of the control input signals (thrust vectoring mode).

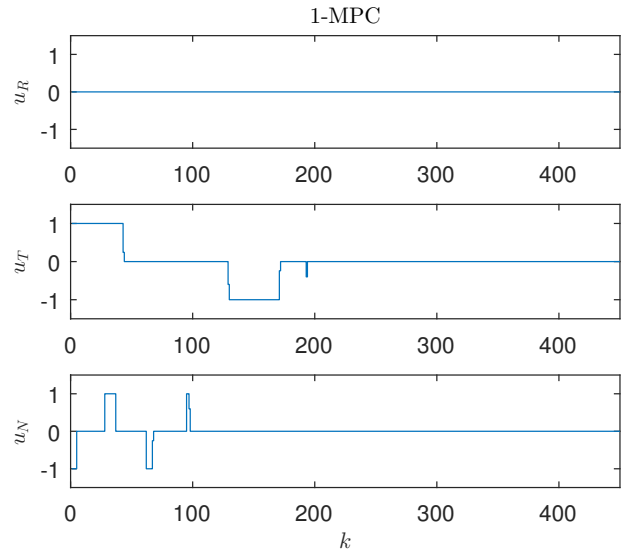


Fig. 4. Control inputs provided by 1-MPC (orthogonal thrusters mode).

replacing (2) with the quadratic cost function

$$\sum_{j=0}^{N-1} [\hat{x}_k^T(j) Q^T Q \hat{x}_k(j) + \hat{u}_k^T(j) \hat{u}_k(j)] + \hat{x}_k^T(N) \Psi \hat{x}_k(N)$$

and considering both cases $q = 2$ and $q = \infty$ in the input constraints (5). The weighting matrix Q is set as in (32) for all controllers. In order to find a suitable horizon length N , the fuel consumption of the control system over the entire simulation has been computed for different values of this parameter. It is observed that fuel consumption decreases with increasing values of N until $N = 180$, while it remains stationary for larger values of N . Based on this result, a value of $N = 192$ has been selected, corresponding to three orbital periods. Such a long horizon does not pose particular computational challenges, because the considered dynamics are relatively slow (the sampling time is in the order of minutes). Indeed, the MPC optimization problems, solved

TABLE II
FUEL CONSUMPTION

Control mode	2-MPC	Q-MPC ($q = 2$)
Thrust vectoring	87.2	93.1

Control mode	1-MPC	Q-MPC ($q = \infty$)
Orthogonal thrusters	105.3	114

using the package CVX [24] and the commercial solver Gurobi, require a computation time of approximately 0.5 s on a 2.6 Ghz CPU. The 2-norm of the state trajectories resulting from the simulation is depicted in Fig. 2. It can be seen that both the 2-MPC and 1-MPC schemes, whose transient response is very similar, achieve finite time convergence. This is a feature which is often observed for MPC laws based on a nonsmooth performance index. On the other hand, the Q-MPC scheme achieves only asymptotic tracking. Moreover, it presents some slowly damped oscillations about the set-point. These can be attributed to the fact that the quadratic cost flattens out near the origin.

The 2-norm of the Q-MPC and 2-MPC input signals are displayed in Fig. 3, for the thrust vectoring scenario. As opposed to Q-MPC, the 2-MPC design results in a sparse control input 2-norm, which represents the actual thrust in this scenario. As shown in Fig. 4, the 1-MPC scheme achieves sparsity on each individual input channel, which represents the actual thrust of each engine in the case of orthogonal thrusters. The fuel consumption (measured as $\sum_k \|u(k)\|_2$ or $\sum_k \|u(k)\|_1$, depending on the control mode) is compared in Table II. As expected, the 2-MPC and 1-MPC schemes deliver the highest fuel efficiency when measured by the 2-norm and the 1-norm, respectively. Remarkably, this is achieved in spite of a faster transient response compared to that of Q-MPC, see Fig. 2. The obtained results clearly demonstrate the suitability of 2-MPC and 1-MPC techniques for spacecraft maneuvering applications. Since the resulting input signal profile is almost on-off, the proposed design may also be exploited for the synthesis of efficient pulse-width-modulation controllers, using, for instance, the method in [25]. This is relevant to the considered problem because many spacecraft engines cannot be throttled.

V. CONCLUSIONS

Two vector norm based predictive control schemes have been derived for spacecraft translational maneuvering. The proposed design allows one to trade off the fuel consumption of the propulsion system with tracking performance, in two different spacecraft maneuvering modes. The resulting closed-loop system response shows both finite time convergence and control sparsity, which are two desirable properties in this class of applications. A more thorough theoretical characterization of these properties is the subject of ongoing research. The optimization of the control design parameters is also under investigation.

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