

Explicit Model Predictive Control Approach for Low-Thrust Spacecraft Proximity Operations

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The key role of autonomous systems in future space missions has made model predictive control a very attractive guidance and control technique. However, the capability of low-power spacecraft processors to handle the real-time computational load of this technique still needs to be fully established, especially for complex control problems. This paper introduces a method to improve the computational efficiency of model predictive control when applied to the problem of autonomous rendezvous and proximity maneuvering using low-thrust propulsion. To ensure safe trajectories in this scenario, a long control horizon is required and the control problem must be solved at a relatively fast sampling rate. The proposed design addresses such requirements by parameterizing the thrust profile with a set of Laguerre functions. In this setting, the number of control variables can be made significantly smaller than the length of the control horizon, as opposed to standard design methods. By exploiting this property in combination with multi-parametric programming techniques, an explicit control law is derived that is suitable for real-time implementation on simple hardware. The performance of this approach is demonstrated on a small spacecraft mission and compared with that of other control techniques.

I. Introduction

The development of guidance and control techniques for spacecraft formation flying is the subject of significant research efforts, due to the key role of such problems in many present and future space missions. Examples include technology demonstrators like PRISMA [1] and PROBA-3 [2], the space interferometer DARWIN [3], the Mars sample return scientific mission [4], and on-orbit servicing projects such as the Automated Transfer Vehicle [5] or the orbital life extension vehicle SMART-OLEV [6].

Of particular interest in this field is the optimization of low-thrust formation flying trajectories, motivated by the application of miniaturized or high-efficiency propulsion technologies [7–10]. When two or more spacecraft in a formation are required to operate in close proximity, these trajectories must be safe with respect to collisions and other possible anomalies [11]. This generally leads to complex trajectory optimization problems, subject to both thrust magnitude and path constraints. Due to the increasing level of autonomy of future space applications, it is critical to compute the solution to these problems in real-time and to design a control system tracking the resulting trajectories [12, 13]. To this purpose, efficient guidance and control algorithms have to be devised. More specifically, this paper tackles the problem of developing an optimal guidance and control scheme for autonomous rendezvous and proximity maneuvering using low-thrust propulsion, in the presence of collision avoidance, thruster plume impingement and line of sight (LoS) constraints.

A wide variety of open-loop guidance techniques have been proposed in the literature for the design of low-thrust rendezvous trajectories, based on either direct or indirect optimization methods [14–16]. These techniques are known to provide accurate numerical solutions, but they cannot cope with the high degree of autonomy required by applications in which disturbance rejection and robustness with respect to perturbations are of primary concern. To circumvent this issue, feedback guidance and control algorithms, with the ability to systematically handle thrust magnitude and path constraints, are commonly used. In particular, model predictive control (MPC), based

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on computing the optimal control sequence over a finite number of future sampling instances, is becoming increasingly attractive [17–20]. In low-thrust problems, however, a long control horizon is needed to guarantee adequate performance, due to the limited control authority provided by the actuators. During close proximity operations, this is coupled with the requirement to use a small discretization step, to avoid the violation of path constraints between discrete time samples. In such cases, the main drawback of MPC is the requirement to solve a trajectory optimization problem with a large number of decision variables at each time sample, which may make this method too computationally intensive to be implemented on-line on low-power spacecraft processors [21].

A possible way of enhancing MPC to overcome this last difficulty is to parameterize the control sequence with a set of Laguerre functions, where the poles of these functions are used to reflect the time scale of the control system, see, e.g., [22]. In this setting, which belongs to the family of direct optimization methods, the number of decision variables can be made significantly smaller than the length of the control horizon, while path constraints can still be enforced over a sufficiently fine discretization grid.

Another important factor, which may prevent the implementation of the MPC design methods discussed so far, is the requirement to embed a control solver with guaranteed runtime on board the spacecraft. This requirement can be avoided by solving the control problem explicitly, i.e. by finding off-line a feedback control law defined on a partition of the state space [23]. In the standard MPC framework, however, this is generally feasible only for low-dimensional problems, due to the exponential growth of the number of regions in the partition with the length of the control sequence [24]. An alternative approach, based on the explicit solution of a quadratically constrained linear quadratic regulator (LQR) problem, has been recently proposed in [25] for a rendezvous problem with thrust constraints, which confirms the need for computationally efficient feedback control methods specifically tailored to the considered application area.

The contribution of this paper is twofold. First, a low-complexity MPC scheme is developed for the low-thrust rendezvous and proximity maneuvering problem. In the derivation of the control algorithm, the trajectory optimization problem is reformulated by parameterizing the control sequence by a set of Laguerre functions, which allows a long control horizon to be realized without

using a large number of decision variables. Then, an explicit control law is derived by exploiting this new algorithm in combination with multi-parametric programming techniques. Such design provides a trade-off between feasibility and performance of the guidance and control system. Since on-line optimization is not required, the novel control law is especially suitable for real-time implementation on board small spacecraft with limited computational capabilities. A detailed simulation-based assessment of the performance achievable with this design is given for an example cubesat mission using a miniaturized electric propulsion system, in comparison to standard MPC and LQR techniques.

The paper is organized as follows. In section II, proximity operations, including terminal rendezvous and docking, are briefly described. Section III then details the main features of the control problem and presents the novel control law, and Section IV illustrates the formation flying model used to validate the proposed approach. The performance of the control law is evaluated through numerical simulations in Section V. Section VI gives some concluding remarks.

II. Problem Setting

The considered problem is that of autonomous rendezvous and proximity operations between two spacecraft in a leader-follower formation, where the attitude of both spacecraft is actively controlled and the leader is not maneuvering. Based on relative position measurements from differential global positioning system (GPS) and optical sensors, the follower spacecraft is required to maintain visual contact and dock with the leader, using low-thrust propulsion. The control objective is to minimize a combination of the fuel expenditure and the time of flight of the maneuver [26], subject to the following requirements to ensure safe trajectories [11].

- Collision avoidance: the spacecraft must not collide with each other.
- LoS: the relative motion must be confined within a certain region of the state space (a cone) to maintain visual contact.
- Plume impingement: the magnitude and/or the amount of thruster firings directed towards the leader must be minimized during the final phase of the approach.

In addition, thrust magnitude and direction constraints must be taken into account in the control problem.

In this paper, vector and matrices are denoted by boldface symbols and $\mathbf{1}$ denotes a vector whose components are all equal to 1, the identity matrix is denoted by $\mathbf{I}$ and the symbol $\mathbf{0}$ denotes the null matrix or vector of compatible dimensions. The symbol $\oplus^n \mathbf{A}$ denotes a block-diagonal matrix with n diagonal blocks, each equal to $\mathbf{A}$ and the norm of a vector is denoted by $\|\cdot\|_n$, where the ∞ , 1 and 2-norms are used. Moreover, the relative motion of the formation is expressed in a rotating local-vertical-local-horizontal (LVLH) frame centered at the leader spacecraft center of mass. The Z axis points towards the Earth's center of mass, the Y axis is aligned with the negative orbit normal and the X axis completes an orthogonal right-handed coordinate system, as illustrated in Fig. 1. In a circular orbit, the X axis is aligned with the spacecraft velocity vector. The X , Y and Z directions are referred to as the along-track, cross-track and radial directions respectively. The XY and the XZ planes are referred as the horizontal-plane and the in-plane and the relative position vector is denoted by

$$\delta \mathbf{r} = [x \ y \ z]^T, \quad (1)$$

where x , y and z are the along-track, cross-track and radial components respectively.

The following assumption are made on the configuration of the formation: (i) the leader orbit is nearly circular, (ii) the distance between the two spacecraft is small compared to the orbit radius and (iii) differential perturbations are negligible. Under these assumptions, the relative motion dynamics are well approximated by the linearized Hill-Clohessy-Wiltshire (HCW) equations [27]

$$\begin{aligned} \ddot{x} &= 2\omega \dot{z} + \frac{u_1}{m} \\ \ddot{y} &= -\omega^2 y + \frac{u_2}{m} \\ \ddot{z} &= 3\omega^2 z - 2\omega \dot{x} + \frac{u_3}{m}, \end{aligned} \quad (2)$$

where u_1 , u_2 and u_3 are the control forces of the follower, expressed in the LVLH frame, m is the mass of the spacecraft, and ω is the LVLH rate.

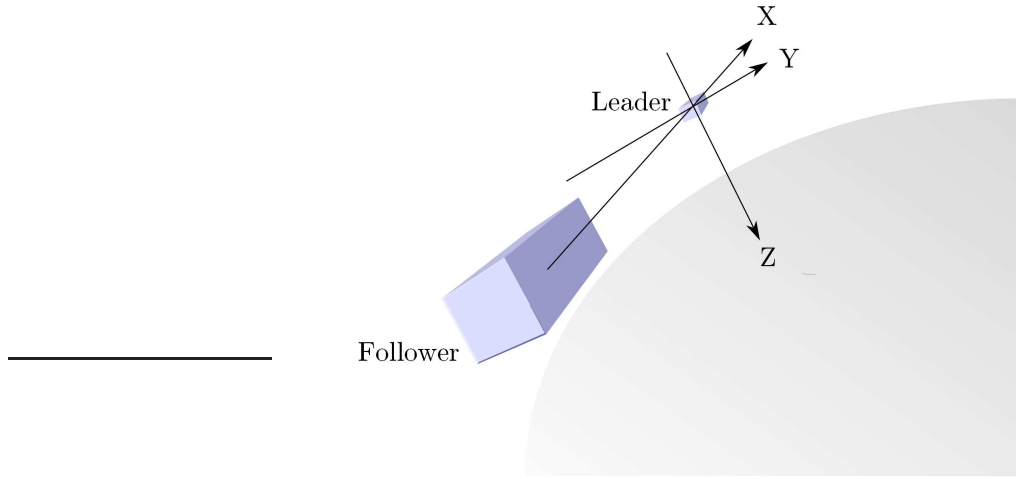


Figure 1. LVLH reference frame.

Moreover, it is assumed that: (iv) both the leader and follower spacecraft are three-axis stabilized to maintain the LVLH attitude, (v) the docking port is located behind the leader and (vi) the propulsion system of the follower can produce thrust only in the along-track and cross-track directions. The position of the docking port can be expressed in terms of relative states as

$$\delta \mathbf{r}_d = [x_d \ 0 \ 0]^T, \quad (3)$$

where $x_d \leq 0$ is fixed. Since radial thrust is not available, $u_3 = 0$ in (2) and the input vector is defined as

$$\mathbf{u} = [u_1 \ u_2]^T. \quad (4)$$

In this setting, any arbitrary initial state $\delta \mathbf{r}(t_0)$ can be steered to $\delta \mathbf{r}_d$, since the in-plane motion in (2) is controllable with the scalar input u_1 [28]. The tracking error is denoted by

$$\mathbf{x} = [x_1 \ \dots \ x_6]^T = [(\delta \mathbf{r} - \delta \mathbf{r}_d)^T \ (\delta \dot{\mathbf{r}} - \delta \dot{\mathbf{r}}_d)^T]^T. \quad (5)$$

where $\delta \dot{\mathbf{r}}_d = \mathbf{0}$, since $\delta \mathbf{r}_d$ is fixed according to (3).

III. Formation Control

Let $\mathbb{U}$ be an admissible input set, $\mathbb{X}$ an admissible subset of the state space defined by path constraints and $J(\mathbf{x}, \mathbf{u})$ a given cost function, defined over the time interval $t \in [t_0, t_f]$. In the considered problem, the input set is bounded by the maximum thrust u_M that can be delivered by the propulsion system, as

$$\mathbb{U} = \{\mathbf{u} : \|\mathbf{u}(t)\|_\infty \leq u_M\}. \quad (6)$$

Collision avoidance and LoS requirements can be expressed as the path constraints

$$\mathbb{X} = \left\{ \mathbf{x} : x_1(t) \leq 0, \sqrt{x_2(t)^2 + x_3(t)^2} \leq -x_1(t) \tan(\theta/2) \right\}, \quad (7)$$

where θ is the field of view of the optical sensor on board the follower spacecraft. For rendezvous and docking of a leader-follower spacecraft pair, a relevant cost function is

$$J(\mathbf{x}, \mathbf{u}) = \alpha \int_{t_0}^{t_f} \|\mathbf{u}(t)\|_1 dt + (1 - \alpha) \int_{t_0}^{t_f} 1 dt + \beta \int_{t_0}^{t_f} \epsilon(t) dt, \quad (8)$$

where the final time t_f is free, $\alpha \in [0, 1]$ is a relative weight on the fuel consumption (first term) and the maneuver time (second term), and $\beta \geq 0$ is a weight on the function ϵ , which accounts for plume impingement effects. If (7) is satisfied, the thruster plume impingement function can, as justified in [29], be taken as

$$\epsilon(t) = \begin{cases} u_1^-(t) & \text{if } -x_1(t) \leq x_{\epsilon 1}, |x_2(t)| \leq x_{\epsilon 2} \text{ and } |x_3(t)| \leq x_{\epsilon 3} \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

where $u_1^-(t)$ is the negative part of the along-track thrust and $x_{\epsilon 1}$, $x_{\epsilon 2}$ and $x_{\epsilon 3}$ are predefined positive constants.

Given $\mathbf{x}(t_0)$, the formation control problem can be stated as

$$\begin{aligned}
& \min_{\mathbf{u}} J(\mathbf{x}, \mathbf{u}) \\
& \text{s.t.} \quad (2) \\
& \mathbf{x} \in \mathbb{X}, \mathbf{u} \in \mathbb{U} \\
& \mathbf{x}(t_f) = \mathbf{0}.
\end{aligned} \tag{10}$$

The problem defined by (10) does not admit a closed-form solution and must be solved numerically. Moreover, it consists of a nonlinear nonsmooth optimization problem, whose on-line solution on board spacecraft with limited computational capabilities may not be possible. For this reason, a number of suboptimal policies have been considered in the literature [17–20, 25, 30, 31].

MPC is an attractive design method for the problem described above, since it enables constraints to be enforced on the system state and on the control inputs. Moreover, the resulting control law can be explicitly parameterized in feedback form. In linear MPC, the endpoint equality constraint $\mathbf{x}(t_f) = \mathbf{0}$ is typically replaced by a weight $\mathbf{W}_f$ on the terminal state of the system, the set $\mathbb{X}$ is approximated by a polyhedral set $\bar{\mathbb{X}}$, and the problem is solved over a finite horizon $T_p = (t_f - t_0)$. Following this approach, problem (10) is reformulated as

$$\begin{aligned}
& \min_{\mathbf{u}} J_c(\mathbf{x}, \mathbf{u}) = \|\mathbf{W}_f \mathbf{x}(t_f)\|_n^n + \int_{t_0}^{t_f} \|\mathbf{W} \mathbf{x}(t)\|_n^n + \|\mathbf{K} \mathbf{u}(t)\|_n^n dt \\
& \text{s.t.} \quad (2) \\
& \mathbf{x} \in \bar{\mathbb{X}}, \mathbf{u} \in \mathbb{U},
\end{aligned} \tag{11}$$

where $\mathbf{W}_f$, $\mathbf{W}$ and $\mathbf{K}$ are square weighting matrices, $\mathbf{K}$ is nonsingular, and values of $n = 1$ or $n = 2$ are considered. Problem (11) is solved under the receding horizon principle to yield a feedback control law which renders the equilibrium point $\mathbf{x} = \mathbf{0}$ asymptotically stable [32]. To ensure an acceptable computational complexity, the non-convex plume impingement function (9) is not included in (11). This approximation turns out to be reasonable for a quadratic performance index ($n = 2$), since in this case the thrust magnitude vanishes close to the steady state, but not for a linear one ($n = 1$), due to the bang-bang structure of the corresponding optimizer [33]. Hence, in the

following it is assumed that $n=2$.

A low-complexity, explicit solution to problem (11) is sought. It is known that, in the worst case, the number of state space regions over which an explicit control law is defined grows exponentially with length of the input sequence [24]. On the other hand, a short input sequence can lead to poor performance or even unfeasibility. A trade-off between computational burden and performance requirements can be made by parameterizing the input sequence via a set of Laguerre functions [22, 34], as described next.

A. MPC design

Using the linearized HCW equations (2), the tracking error dynamics are represented by the state space model

$$\dot{\mathbf{x}} = \mathbf{A}_c \mathbf{x} + \mathbf{B}_c \mathbf{u}, \quad (12)$$

with

$$\mathbf{A}_c = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2\omega \\ 0 & -\omega^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3\omega^2 & -2\omega & 0 & 0 \end{bmatrix} \quad (13)$$

and

$$\mathbf{B}_c = \begin{bmatrix} 0 & 0 & 0 & 1/m & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/m & 0 \end{bmatrix}^T. \quad (14)$$

For digital implementation of the control law, the system is discretized with a sampling period T_s using a zero-order hold, resulting in the discrete state space model

$$\mathbf{x}(k+1) = \mathbf{A} \mathbf{x}(k) + \mathbf{B} \mathbf{u}(k), \quad (15)$$

where

$$\mathbf{A} = e^{\mathbf{A}_c T_s}, \quad \mathbf{B} = \left(\int_0^{T_s} e^{\mathbf{A}_c \tau} d\tau \right) \mathbf{B}_c. \quad (16)$$

The MPC design requires the predicted future states for a number of steps ahead, where these are generated from the state space model (15) at the current sampling instant based on the current state and the computed input sequence. Let $\mathbf{u}(k+j)$ denote the input to be computed j sampling steps ahead from the current sampling instant k . The basic idea underpinning Laguerre MPC (LMPC) is to parameterize $\mathbf{u}(k+j)$ using a set of discrete Laguerre polynomials, as

$$\mathbf{u}(k+j) = \begin{bmatrix} u_1(k+j) \\ u_2(k+j) \end{bmatrix} \approx \begin{bmatrix} \mathbf{l}_1^T(j) & \mathbf{0} \\ \mathbf{0} & \mathbf{l}_2^T(j) \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \mathbf{L}(j) \boldsymbol{\eta}, \quad (17)$$

where $\mathbf{l}_i(j)$ is the Laguerre function vector and $\boldsymbol{\eta}$, which represents the new decision vector, is termed the coefficient vector.

The Laguerre function vector satisfies the difference equation

$$\mathbf{l}_i(j+1) = \begin{bmatrix} a_i & 0 & \dots & \dots & 0 \\ b_i & a_i & \ddots & \vdots & 0 \\ -a_i b_i & b_i & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -a_i^{N_i-2} & -a_i^{N_i-3} b_i & \dots & b_i & a_i \end{bmatrix} \mathbf{l}_i(j) \quad (18)$$

with

$$\mathbf{l}_i(0) = \sqrt{b_i} \begin{bmatrix} 1 & -a_i & a_i^2 & -a_i^3 & \dots & (-1)^{N_i-1} a_i^{N_i-1} \end{bmatrix}^T, \quad (19)$$

where $b_i = (1 - a_i^2)$, N_i is the number of terms in the expansion and $a_i \in [0, 1]$ is the scaling factor of the Laguerre network for input u_i . Both a_i and N_i are fixed design parameters. Setting $a_i = 0$ in (18) and (19), and using (17), give that

$$\mathbf{l}_i^T(j) \boldsymbol{\eta}_i = \begin{cases} u_i(k+j) & \forall j \in \{0 \dots N_i\} \\ 0 & \forall j > N_i, \end{cases} \quad (20)$$

which corresponds to the standard MPC design with control horizon N_i . Choosing $a_i > 0$ allows a trade-off to be made between the accuracy the approximation (17) and the time scale of the control trajectory, i.e., $\mathbf{l}_i^T(j) \boldsymbol{\eta}_i$ exponentially decays to zero instead of being identically zero for $j > N_i$. This is especially relevant when the number of decision variables N_i is selected to be small to keep the computation feasible and then the truncated parametrization given by (20) cannot adequately describe the future input trajectory. Substituting (17) into (15), the state dynamics N_p sampling instants ahead of k are given by

$$\begin{cases} \mathbf{x}(k+1|k) = \mathbf{A} \mathbf{x}(k) + \mathbf{B}\mathbf{L}(0) \boldsymbol{\eta} \\ \mathbf{x}(k+2|k) = \mathbf{A}^2 \mathbf{x}(k) + (\mathbf{A}\mathbf{B}\mathbf{L}(0) + \mathbf{B}\mathbf{L}(1)) \boldsymbol{\eta} \\ \vdots \\ \mathbf{x}(k+N_p|k) = \mathbf{A}^{N_p} \mathbf{x}(k) + (\mathbf{A}^{N_p-1} \mathbf{B}\mathbf{L}(0) + \dots + \mathbf{B}\mathbf{L}(N_p-1)) \boldsymbol{\eta} . \end{cases} \quad (21)$$

where the prediction horizon N_p is unrelated to the number of components of $\boldsymbol{\eta}$, which is equal to $(N_1 + N_2)$.

The prediction model can be written in the compact form

$$\mathbf{X} = \mathbf{F} \mathbf{x}(k) + \boldsymbol{\Phi} \boldsymbol{\eta} , \quad (22)$$

where

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} \mathbf{x}^T(k+1|k) & \mathbf{x}^T(k+2|k) & \dots & \mathbf{x}^T(k+N_p|k) \end{bmatrix}^T \\ \mathbf{F} &= \begin{bmatrix} (\mathbf{A})^T & (\mathbf{A}^2)^T & \dots & (\mathbf{A}^{N_p})^T \end{bmatrix}^T \\ \boldsymbol{\Phi} &= \begin{bmatrix} \mathbf{B}\mathbf{L}(0) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{A}\mathbf{B}\mathbf{L}(0) & \mathbf{B}\mathbf{L}(1) & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{A}^{N_p-1} \mathbf{B}\mathbf{L}(0) & \mathbf{A}^{N_p-2} \mathbf{B}\mathbf{L}(1) & \dots & \mathbf{B}\mathbf{L}(N_p-1) \end{bmatrix} . \end{aligned} \quad (23)$$

Moreover, the cost function (11) is discretized for $n = 2$ and $N_p = T_p/T_s$, to give

$$J_d = \mathbf{X}^T \mathbf{Q} \mathbf{X} + \boldsymbol{\eta}^T \mathbf{R} \boldsymbol{\eta}, \quad (24)$$

where for the remainder of this paper $\mathbf{W}_f = \mathbf{W}$, $\mathbf{Q} = \oplus^{N_p} T_s \mathbf{W}^T \mathbf{W}$ is a $6N_p \times 6N_p$ matrix and $\mathbf{R} = T_s \mathbf{M}_u^T (\oplus^{N_p} \mathbf{K}^T \mathbf{K}) \mathbf{M}_u$ is a $(N_1 + N_2) \times (N_1 + N_2)$ matrix, with

$$\mathbf{M}_u = \begin{bmatrix} \mathbf{L}^T(0) & \mathbf{L}^T(1) & \dots & \mathbf{L}^T(N_p - 1) \end{bmatrix}^T. \quad (25)$$

Hence, the minimization of (24) can be equivalently rewritten as

$$\min_{\boldsymbol{\eta}} \boldsymbol{\eta}^T \boldsymbol{\Omega} \boldsymbol{\eta} + 2 \mathbf{x}^T(k) \boldsymbol{\Psi}^T \boldsymbol{\eta}, \quad (26)$$

where $\boldsymbol{\Omega} = (\boldsymbol{\Phi}^T \mathbf{Q} \boldsymbol{\Phi} + \mathbf{R})$ and $\boldsymbol{\Psi} = \boldsymbol{\Phi} \mathbf{Q} \mathbf{F}$.

In the absence of constraints, the global minimum of problem (26) is attained (assuming the required matrix inverse exists) at

$$\boldsymbol{\eta}^*(k) = -\boldsymbol{\Omega}^{-1} \boldsymbol{\Psi} \mathbf{x}(k). \quad (27)$$

According to the receding horizon principle, only the first element of the optimal input sequence is applied to the plant, so that

$$\mathbf{u}(k) = \mathbf{L}(0) \boldsymbol{\eta}^*(k). \quad (28)$$

Input and state constraints are included in the MPC design to account for the operating range of the actuators and to ensure safe proximity operations. Unlike the unconstrained case, the constrained MPC problem does not admit an analytic solution and must be solved numerically. For a given set of samples $\mathbb{M}_u$, on which input constraints are enforced, (6) can be rewritten as

$$-u_M \mathbf{1} \leq \mathbf{L}(j) \boldsymbol{\eta} \leq u_M \mathbf{1} \quad \forall j \in \mathbb{M}_u \subseteq \{0, \dots, N_p - 1\}. \quad (29)$$

To reduce the sensitivity of the control system to measurement noise, one possibility is to introduce a slack variable $s_1 \geq 0$, which weights the variation of the control input, and penalize it in the cost

function. The value of s_1 is obtained from the following linear inequalities:

$$\begin{aligned} -s_1 \mathbf{1} &\leq \mathbf{L}(0) \boldsymbol{\eta} - \mathbf{u}(k-1) \leq \mathbf{1} s_1 & j = 0 \\ -s_1 \mathbf{1} &\leq (\mathbf{L}(j) - \mathbf{L}(j-1)) \boldsymbol{\eta} \leq \mathbf{1} s_1 & \forall j \in \{1, \dots, N_p - 1\}, \end{aligned} \quad (30)$$

where $\mathbf{u}(k-1)$ is treated as an additional input to the optimization problem. The nonlinear path constraints (7) are approximated by the following linear inequalities:

$$\begin{aligned} \mathbf{C} \mathbf{x}(k+j|k) &\leq \mathbf{1} s_2 + \mathbf{d} \quad \forall j \in \mathbb{M}_x \subseteq \{1, \dots, N_p\} \\ \mathbf{C} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \tan(\theta/2)/\sqrt{2} & 1 & 0 & 0 & 0 & 0 \\ \tan(\theta/2)/\sqrt{2} - 1 & 0 & 0 & 0 & 0 & 0 \\ \tan(\theta/2)/\sqrt{2} & 0 & 1 & 0 & 0 & 0 \\ \tan(\theta/2)/\sqrt{2} & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \\ \mathbf{d} &= [d_1 \quad \mathbf{1}^T \boldsymbol{\varepsilon}]^T, \end{aligned} \quad (31)$$

where $d_1, \boldsymbol{\varepsilon} \geq 0$ are given tolerances and $s_2 \geq 0$ is a slack variable which relaxes the formulation in the ∞ -norm sense, to retain the feasibility of the problem against possible conflicts between input and state constraints. Assuming $s_2 = 0$, $\boldsymbol{\varepsilon} > 0$ and $d_1 < \sqrt{2} \boldsymbol{\varepsilon} / \tan(\theta/2)$, (31) represents the polyhedral approximation of the LoS cone depicted in Fig. 2, for the set of samples $\mathbb{M}_x$.

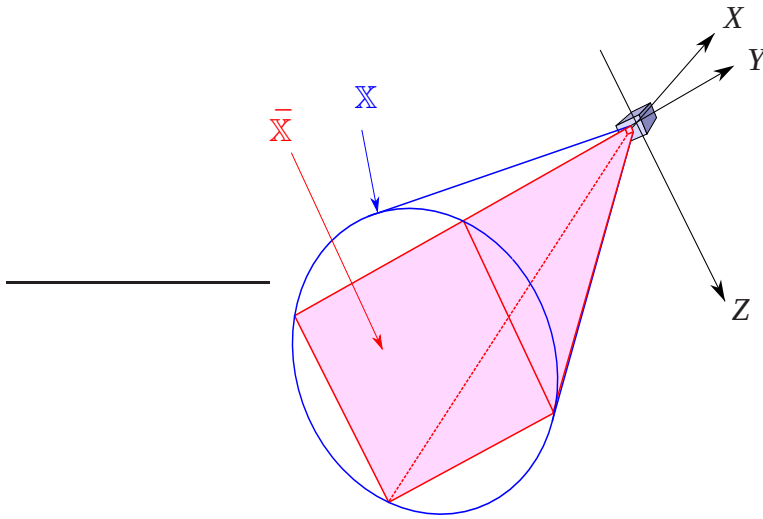


Figure 2. LoS cone approximation.

Let $\eta_C = [\eta^T \ s_1 \ s_2]^T = [\eta^T \ s^T]^T$ be the augmented decision vector and $\mathbf{x}_C(k) = [\mathbf{x}^T(k) \ \mathbf{u}^T(k-1)]^T$ the augmented input in the constrained optimization problem and assume, for notational simplicity, that (29) and (31) are defined for all samples. Then, the constraints (29), (30) and (31) can be written in the compact form

$$\begin{bmatrix} \mathbf{M}_u & \mathbf{0} & \mathbf{0} \\ -\mathbf{M}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_\Delta & -\mathbf{1} & \mathbf{0} \\ -\mathbf{M}_\Delta & -\mathbf{1} & \mathbf{0} \\ \mathbf{C}_N \Phi & \mathbf{0} & -\mathbf{1} \end{bmatrix} \begin{bmatrix} \eta \\ s_1 \\ s_2 \end{bmatrix} \leq \begin{bmatrix} \mathbf{1}u_M \\ \mathbf{1}u_M \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{d}_N \end{bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{U}_\Delta \\ \mathbf{0} & \mathbf{U}_\Delta \\ \mathbf{C}_N \mathbf{F} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{u}(k-1) \end{bmatrix}, \quad (32)$$

where $\mathbf{F}$ and Φ are given by (23), $\mathbf{C}_N = \oplus^{N_p} \mathbf{C}$, $\mathbf{d}_N = [\mathbf{d}^T \dots \mathbf{d}^T]^T$ and

$$\begin{aligned} \mathbf{M}_\Delta &= \begin{bmatrix} \mathbf{L}^T(0) & \mathbf{L}^T(1) - \mathbf{L}^T(0) & \dots & \mathbf{L}^T(N_p - 1) - \mathbf{L}^T(N_p - 2) \end{bmatrix}^T \\ \mathbf{U}_\Delta &= \begin{bmatrix} \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix}^T \end{aligned} \quad (33)$$

The constrained LMPC problem of the form (26) to be solved is

$$\begin{aligned} \min_{\eta_C} \quad & \eta_C^T \begin{bmatrix} \Omega & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_s \end{bmatrix} \eta_C + 2 \mathbf{x}_C^T(k) [\Psi \ \mathbf{0}]^T \eta_C \\ \text{s.t.} \quad & (32) \end{aligned} \quad (34)$$

where $\mathbf{R}_s$ is a 2×2 positive definite diagonal matrix which heavily penalizes the slack vector $\mathbf{s}$. Problem (34) is solved at each sampling instant k , yielding $\eta_C^*(k)$, and the first element of the control sequence is applied to the plant via (28).

B. Explicit Laguerre MPC

Even if the optimization problem (34) can be solved efficiently using existing quadratic programming (QP) algorithms, the required computations may not be feasible for spacecraft with low processing power. Moreover, the execution time of QP solvers is in general not guaranteed, whereas

the reliability of the control system is a primary concern for space applications. In this respect, one possibility is to use explicit MPC.

Before proceeding, it is useful to rewrite the constrained LMPC problem in terms of the simplified notation

$$\begin{aligned} \min_{\eta_C} \quad & \eta_C^T \mathbf{H} \eta_C + 2 \mathbf{x}_C^T(k) \mathbf{G} \eta_C \\ \text{s.t.} \quad & \mathbf{M} \eta_C \leq \mathbf{D} + \mathbf{E} \mathbf{x}_C(k), \end{aligned} \quad (35)$$

where the matrices $\mathbf{H}$, $\mathbf{G}$, $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{E}$ are obtained from (32) and (34). By introducing the vector

$$\mathbf{z} \triangleq \eta_C + \mathbf{H}^{-1} \mathbf{G}^T \mathbf{x}_C(k), \quad (36)$$

problem (35) can be transformed by completing squares into the equivalent multi-parametric quadratic program

$$\begin{aligned} \min_{\mathbf{z}} \quad & \mathbf{z}^T \mathbf{H} \mathbf{z} \\ \text{s.t.} \quad & \mathbf{M} \mathbf{z} \leq \mathbf{D} + (\mathbf{E} + \mathbf{M} \mathbf{H}^{-1} \mathbf{G}^T) \mathbf{x}_C(k), \end{aligned} \quad (37)$$

where $\mathbf{x}(k)$, which appears only in the right hand side of the equation, is treated as a parameter vector.

Problem (37) can be solved explicitly for all the parameters $\mathbf{x}_C(k)$ inside a given polyhedral set $\bar{\mathbb{X}}_C$, as described in, for example, [23]. For the proposed MPC design, it is beneficial to consider a region of additional size $\mathbf{d}_s \geq \mathbf{0}$ with respect to the set defined by (31), together with the maximum excursion of the control. Since the resulting set is not bounded, auxiliary bounds are specified for

the along-track position and the velocity tracking errors using

$$\mathbf{C}_a \mathbf{x}(k) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ k_1 & 0 & 0 & 1 & 0 & 0 \\ k_1 & 0 & 0 & -1 & 0 & 0 \\ k_2 & 0 & 0 & 0 & 1 & 0 \\ k_2 & 0 & 0 & 0 & -1 & 0 \\ k_3 & 0 & 0 & 0 & 0 & 1 \\ k_3 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \mathbf{x}(k) \leq \mathbf{d}_a = \begin{bmatrix} x_M \\ \mathbf{1}\varepsilon_a \end{bmatrix}, \quad (38)$$

where x_M is the maximum feasible along-track separation between the two spacecraft, $\varepsilon_a \geq 0$ is a specified tolerance and k_1 , k_2 and k_3 are positive slopes. The final form of the parameter space is

$$\bar{\mathbb{X}}_C = \left\{ \mathbf{x}_C(k) : \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \\ \mathbf{0} & -\mathbf{I} \\ \mathbf{C}_a & \mathbf{0} \end{bmatrix} \mathbf{x}_C(k) \leq \begin{bmatrix} \mathbf{d} + \mathbf{d}_s \\ \mathbf{1} u_M \\ \mathbf{1} u_M \\ \mathbf{d}_a \end{bmatrix} \right\}. \quad (39)$$

The solution $\mathbf{z}^*(\mathbf{x}_C(k))$ of (37) is a piece-wise affine linear function defined over a polyhedral partition of $\bar{\mathbb{X}}_C$, which can be stored in look-up table form. At each sampling instant, the thrust command is

$$\mathbf{u}(k) = \begin{bmatrix} \mathbf{L}(\mathbf{0}) & \mathbf{0} \end{bmatrix} \left(\mathbf{z}^*(\mathbf{x}_C(k)) - \mathbf{H}^{-1} \mathbf{G}^T \mathbf{x}_C(k) \right), \quad (40)$$

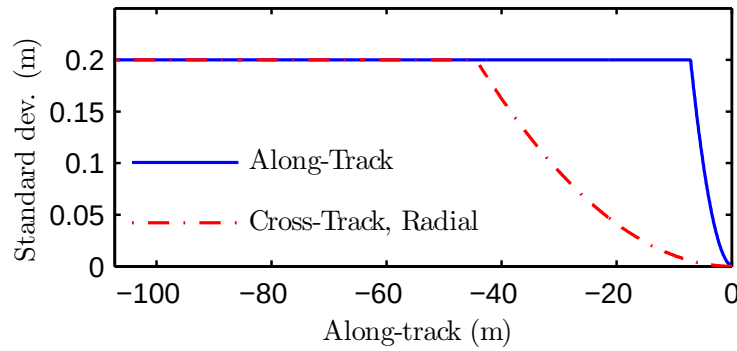
where the on-line computational load is limited to a piece-wise affine function evaluation. This requirement consists of locating the state space region and hence the look-up table entry that contains the pre-computed control law for a given $\mathbf{x}_C(k)$, through the solution of a set-membership problem.

IV. Reference Mission

A possible scenario for the application of the LMPC design developed in this paper is a low Earth orbit formation flying mission performed by two cubesat size spacecraft, where the relative dy-

Table 1. PPT specifications

Mass	180 g (wet mass) + 90 g (electronics)
Dimensions	90.17 x 95.89 x 31 mm
Impulse Bit	40 μ Ns
Pulse frequency	≤ 1 Hz
Total Impulse	42 Ns
Specific Impulse	608 s
Power	0.3-4W
Misalignment	1° per axis
Noise (std. dev.)	5% of the nominal impulse

**Figure 3. Standard deviation of the measurement uncertainty.**

namics are controlled by means of a miniaturized electric propulsion system. At the beginning of the operative phase, the spacecraft are flying in a near circular polar orbit, at an altitude of approximately 450 km. The leader and follower spacecraft have identical physical parameters: the total mass of each of them is 3 kg, the bus size is $30 \times 10 \times 10$ cm³ and the cross-sectional area is 10×10 cm². The electric propulsion system considered in this work is a set of pulsed plasma thrusters (PPT) specifically designed for application to cubesats, as described in [35]. Table 1 gives the characteristics of the PPT model.

The relative position measurement model is based on the specifications of differential GPS and optical sensors [36, 37], where the field of view of the optical sensor is assumed to be $\theta = 30^\circ$. The standard deviation of the measurement uncertainty is given, as a function of the along-track separation between the two spacecraft, in Fig. 3. Relative velocity is estimated from relative position measurements by using a symmetric finite impulse response filter of order 8. Based on

these data, the follower spacecraft is required to dock with the leader whilst satisfying LoS and input constraints, using two pairs of opposite PPTs aligned with the along-track and cross-track directions.

A high-accuracy nonlinear simulation model has been developed for the considered scenario. The state vector of the model includes the position and velocity vectors for the leader ($\mathbf{r}_L, \mathbf{v}_L$) and the follower ($\mathbf{r}_F, \mathbf{v}_F$). The equations which describe the evolution of the state vector in the Earth centered inertial (ECI) frame are

$$\dot{\mathbf{r}}_L = \mathbf{v}_L \quad (41)$$

$$\dot{\mathbf{v}}_L = -\frac{\mu}{r_L^3} \mathbf{r}_L + \mathbf{a}_L \quad (42)$$

$$\dot{\mathbf{r}}_F = \mathbf{v}_F \quad (43)$$

$$\dot{\mathbf{v}}_F = -\frac{\mu}{r_F^3} \mathbf{r}_F + \mathbf{a}_F, \quad \mathbf{v}_F^+ = \mathbf{v}_F^- + \Delta \mathbf{v}_F, \quad (44)$$

where μ is the gravitational parameter of the Earth and (44) accounts for the impulsive change $\Delta \mathbf{v}_F$ of the follower velocity due to PPT operation. The calculation of the disturbance accelerations $\mathbf{a}_L$ and $\mathbf{a}_F$ is based on the main orbital perturbations acting on spacecraft at low altitudes. A spherical harmonic expansion up to degree and order 9 is used for the Earth's gravity field [38]. The drag force is calculated using a drag coefficient of 2.5 and the Jacchia-71 model to approximate the atmospheric density [39]. A cannonball model is employed for the calculation of the solar radiation force, taking into account eclipse conditions. Disturbance accelerations due to the point-mass lunar and solar gravity fields are also considered, where the position of the Sun and Moon is obtained through precise ephemerides. True relative position and velocity are expressed in the LVLH frame as

$$\delta \mathbf{r} = \mathbf{R}_L^I (\mathbf{r}_F - \mathbf{r}_L) \quad (45)$$

$$\delta \dot{\mathbf{r}} = \mathbf{R}_L^I (\mathbf{v}_F - \mathbf{v}_L) - [\omega \times] \mathbf{R}_L^I (\mathbf{r}_F - \mathbf{r}_L), \quad (46)$$

where $\mathbf{R}_L^I$ is the matrix that represents the coordinate transformation between the inertial and LVLH

frames and $[\omega \times]$ is the skew-symmetric matrix of $\omega = [0 \ \omega \ 0]^T$. The LVLH rate is given by

$$\omega = -\sqrt{\frac{\mu}{\|\mathbf{r}_L\|_2^3}}. \quad (47)$$

An integral pulse frequency modulator is used to convert the continuous control signal from the control algorithm into discrete pulses of fixed magnitude, as required by PPT operation. The modulator delivers a pulse p_i , on input channel i , whenever the integral of the commanded thrust $U_i(t)$ is greater than or equal to the impulse bit U_M of the thrusters. For each component of the input $\mathbf{u}$, one has

$$p_i(t_k) = \begin{cases} U_M \text{sgn}(U_i(t_k)) & \text{if } |U_i(t_k)| \geq U_M \\ 0 & \text{if } |U_i(t_k)| < U_M, \end{cases} \quad (48)$$

where

$$U_i(t_k) = U_i(t_{k-1}) + \frac{u_i(t_{k-1}) + u_i(t_k)}{2} \Delta t - p_i(t_{k-1}), \quad (49)$$

$\Delta t = t_k - t_{k-1}$ and $i = 1, 2$. Under the assumption that the attitude of the spacecraft is controlled to match the orientation of the LVLH frame, the impulsive velocity change, expressed in the inertial frame, is given by

$$\Delta \mathbf{v}_F = \mathbf{R}_I^L \delta \mathbf{R} \frac{\mathbf{p} + \mathbf{w}}{m}, \quad (50)$$

where $\mathbf{R}_I^L = (\mathbf{R}_L^I)^T$, $\mathbf{p} = [p_1 \ p_2 \ 0]^T$, $\delta \mathbf{R}$ denotes the thruster alignment error and $\mathbf{w} = [w_1 \ w_2 \ 0]^T$ represents the thruster noise.

V. Simulations

In this section, the results from a simulation case study of the performance of the proposed design are given, including a comparison with standard MPC and LQR techniques and a feasibility assessment for the reference mission. The control law is tuned to trade-off between the maneuver time and the fuel consumption. Notice that, since the error system (12) is null controllable with vanishing input energy [28, 40], the solution to (26) approaches the minimum fuel solution for a sufficiently long prediction horizon and a relatively small state weight compared to the input weight.

Table 2. LMPC tuning parameters

State weight	$\mathbf{W} = \text{diag}(1, 0, 1, 10^5, 3 \cdot 10^5, 10^5)$
Input weight	$\mathbf{K} = \mathbf{I} \cdot 2 \cdot 10^{10}$
Slack weight	$\mathbf{R}_s = \text{diag}(10^{14}, 10^5)$
Sampling time	$T_s = 10 \text{ sec}$
Prediction horizon	$N_p = 1000$
Laguerre terms	$N_1 = N_2 = 4$
Scaling factor	$a_1 = a_2 = 0.67$
Input constraint	$u_M = 40 \mu\text{N}, \mathbb{M}_u = \{0\}$
State constraint	$d_1 = \varepsilon = 2 \text{ cm}, \mathbb{M}_x = \{1, 150\}$

Table 3. Parameters of explicit LMPC

Max. feasible separation	$x_M = 350 \text{ m}$
Additional LoS region	$\mathbf{d}_s = [0.1, 10, 10, 10, 10]^T \text{ m}$
Velocity slopes	$k_1 = 0.002, k_2 = k_3 = 0.001$
Velocity tolerance	$\varepsilon_a = 0.5 \text{ mm/s}$

The elimination of radial thrust, which is an underlying assumption of this work, has proven to be effective in improving the fuel efficiency of control laws based on quadratic performance indices [41]. Since the cross-track motion is a simple undamped oscillatory motion which is decoupled from the rest of the system, pure derivative control can be applied on this axis to provide adequate damping [42]. Hence, the cross-track position weighting is set to zero in (24). The other tuning parameters are selected with a trial-and-error procedure based on numerical simulations. Table 2 summarizes the parameters of the controller.

An explicit control law is computed off-line by solving the multi-parametric quadratic program (37) for the parameter space defined by (39) and the quantities in Table 3. The solution is a polyhedral partition of the parameter space defined by 946 regions in 8 dimensions (6 states for the relative motion plus 2 inputs). The real-time computation of the control law on board the spacecraft is conveniently reduced to a set-membership evaluation and the step size of the integral pulse frequency modulator is taken as $\Delta t = 1 \text{ s}$, according to the thruster specifications in Table 1.

A. Control law comparison

In this section the LMPC control law is compared to an LQR and a standard MPC law with equal state and input weights, by application to the linearized HCW model (2). In this study, the standard MPC formulation is recovered from the LMPC scheme by setting the scaling factors a_1 , a_2 of the Laguerre function network to zero.

Figure 4 gives the results for the three controllers in terms of the magnitude of the tracking error for a sample rendezvous and docking maneuver. As expected, the fastest convergence is achieved by the LQR controller, which does not enforce input and state constraints. Moreover, the LMPC scheme shows a much better transient response than the standard MPC scheme. In particular, the former avoids the oscillatory behavior displayed by the latter (which is due to a “myopic” parametrization of the input sequence). The superior performance of the LMPC scheme is supported by the fact that the explicit solution to the standard MPC problem is defined on a larger number of regions (1015). The horizontal-plane and the in-plane motions are shown in Fig. 5, together with the sections of the pyramid that approximate the LoS cone. Evidently the LQR controller is unable to keep the radial tracking error within the LoS constraints.

Figure 6 gives the thrust profiles calculated by each control law. During the initial phase of the maneuver, the along-track LQR command exceeds the maximum thrust which can be delivered by the propulsion system. Since the magnitude of the input is hard-constrained in the model predictive framework, both the MPC and the LMPC commands do not exceed the maximum operating range of the actuators. The thrust profiles calculated from noisy measurements (see Fig. 3) are reported in Fig. 7. Comparing these results with those in Fig. 6, it is evident that the use of Laguerre functions in combination with an appropriate weight on the input variation provides the lowest sensitivity

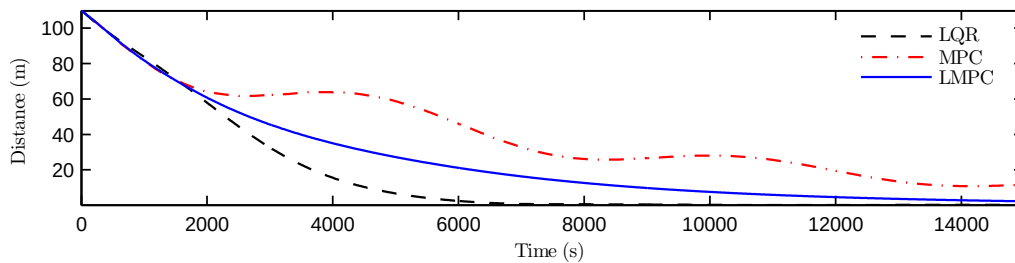


Figure 4. Tracking performance.

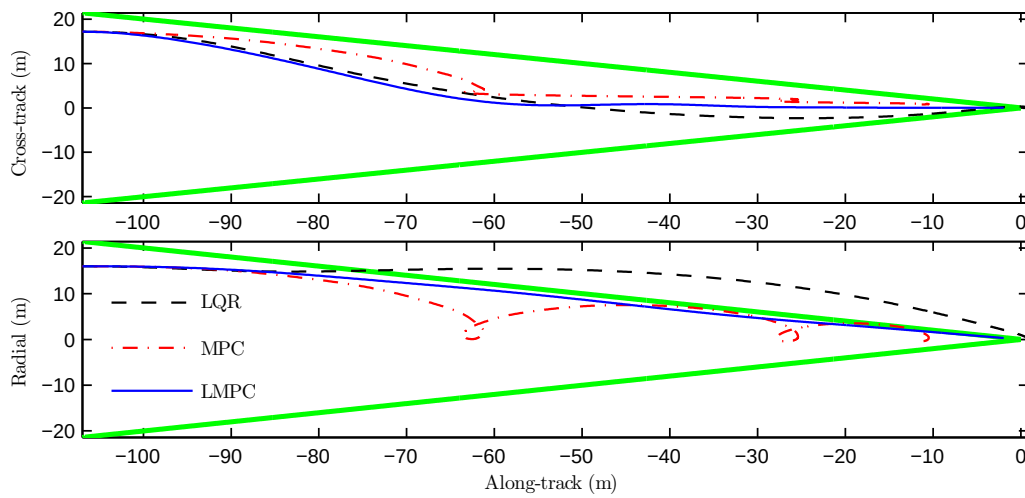


Figure 5. LoS constraints (shown in bold) and relative trajectory.

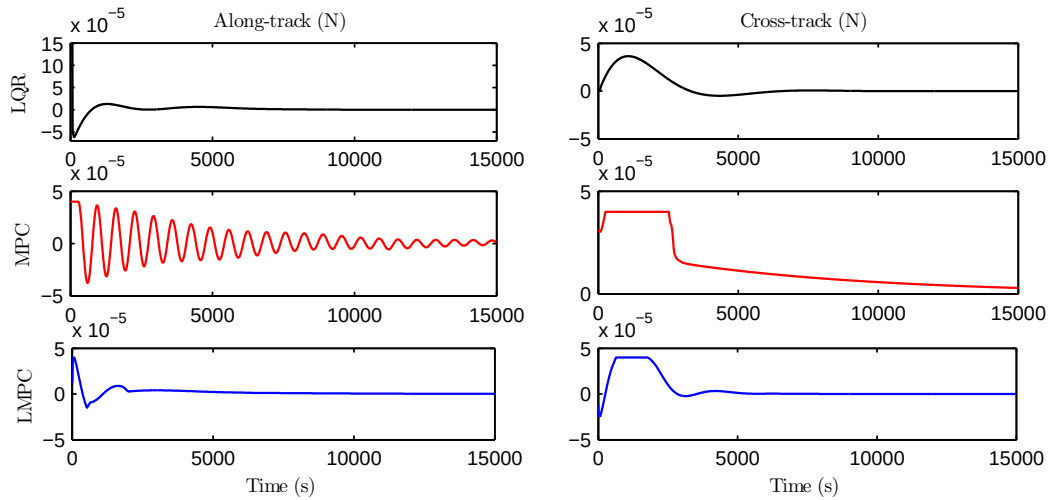


Figure 6. Thrust profile.

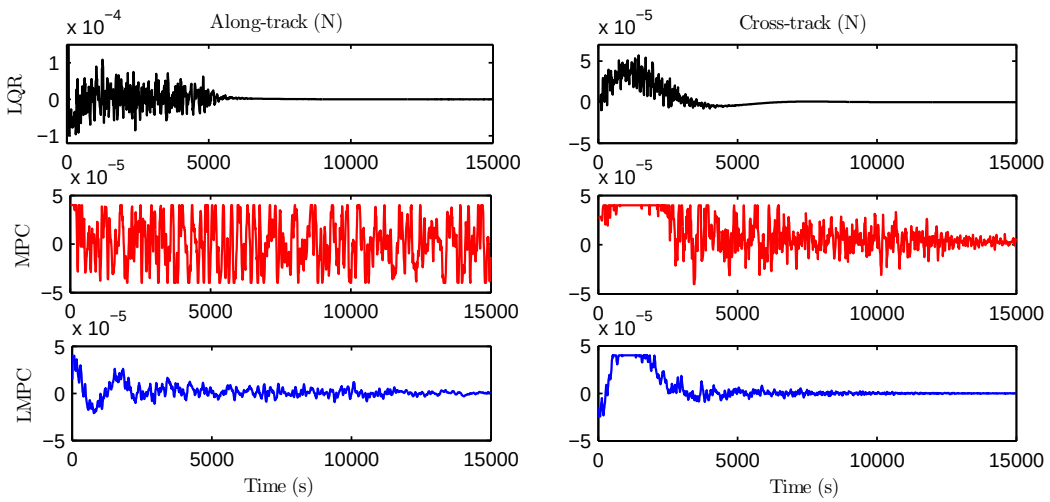


Figure 7. Thrust profile from noisy measurements.

with respect to measurement noise. This is confirmed by Table 4, which reports the total impulse delivered by the control system with and without noise. The increase of the total impulse due to noise is in the order of 60% for both the LQR and the standard MPC schemes, but just 31% for the LMPC design.

Table 4. Total impulse sensitivity to noise

Type	LQR	MPC	LMPC
Without noise	0.146 Ns	0.330 Ns	0.121 Ns
With noise	0.2420 Ns	0.5320 Ns	0.1591 Ns
	+65%	+62%	+31%

B. Docking maneuver simulation

A number of docking maneuvers have been simulated using the nonlinear formation flying model (41)-(44), which includes the PPT model, in combination with the observation model in Fig. 3 and the LMPC control law. The set of initial conditions for which the relative motion lies near the edge of the LoS region has been identified as the worst-case simulation scenario. Two representative simulation cases are reported, with equal along-track initial separation and opposite initial conditions for the cross-track and radial components of the relative position vector. The initial conditions of Case 1 are the same as those used for the comparison of control laws.

Figures 8 and 9 show that the LMPC control law is able to drive the follower spacecraft to the docking position while satisfying the LoS constraints in both cases. The magnitude of the relative position vector at the end of the simulation is equal to 9 mm for Case 1 and 4 cm for Case 2. The good agreement of the Case 1 results with those for the linear simulations (Figs. 4

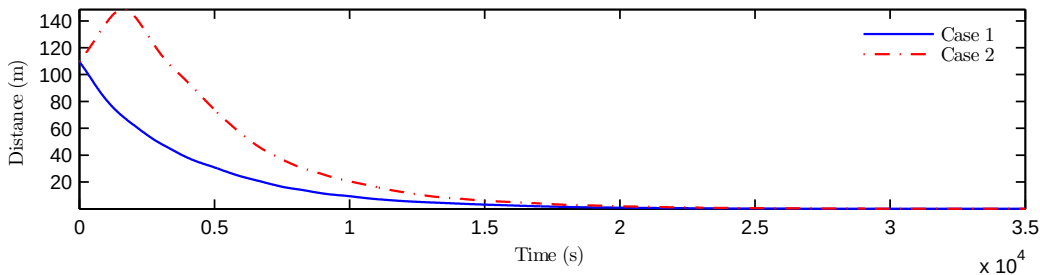


Figure 8. LMPC tracking performance.

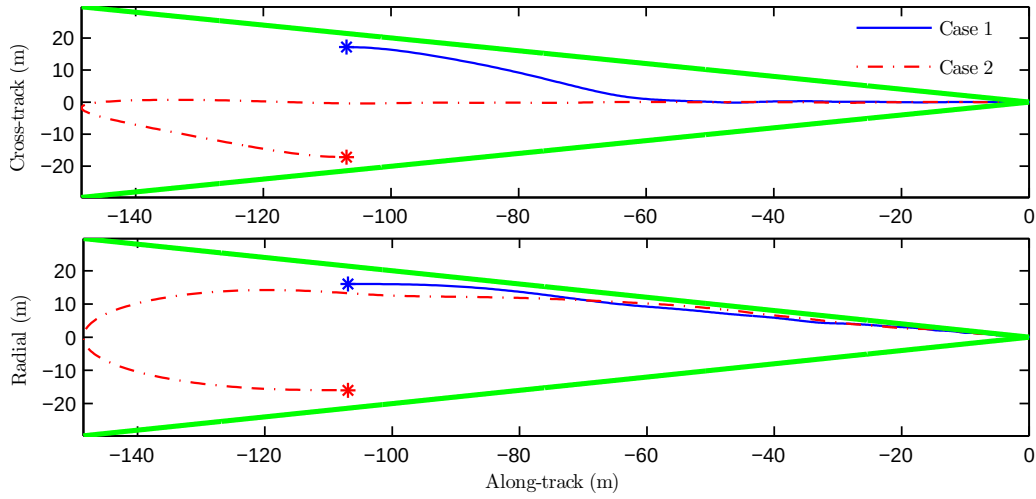


Figure 9. LoS constraints (shown in bold) and LMPC trajectory.

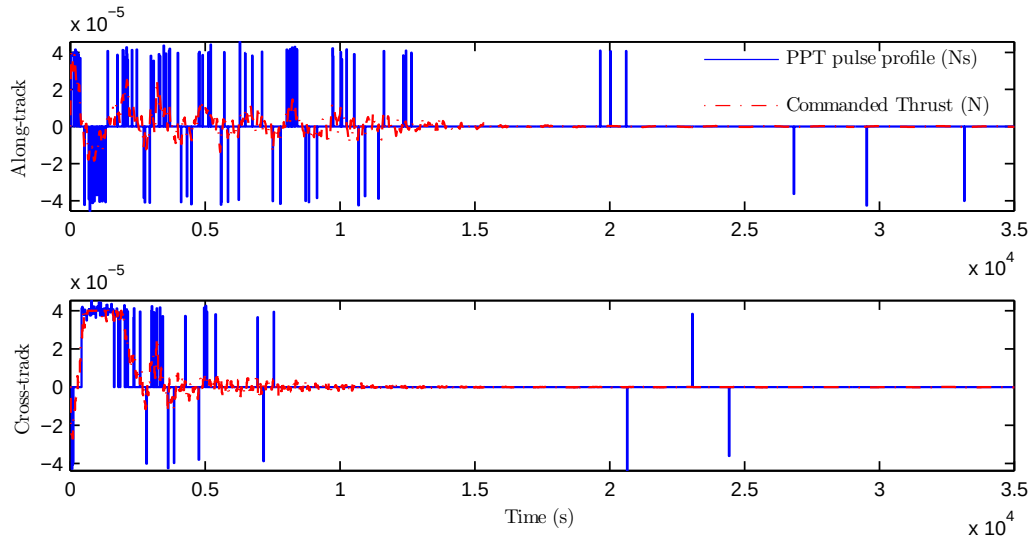


Figure 10. PPT pulse profile and commanded thrust for Case 1.

and 5) suggests an appreciable robustness of the control system with respect to perturbations. The PPT pulse profile is reported in Fig. 10, together with the LMPC command, for Case 1 (similar results were obtained for Case 2). These results show almost no impulses are commanded in the negative along-track direction during the final phase of the approach, which indicates that plume impingement is avoided (see (9)).

As a final comparison, the results presented in this section are evaluated against the fuel-optimal, open-loop solution of problem (10) (obtained for $\alpha = 1$ and $\beta = 0$ in (8)). To enable this comparison, the final time t_f in (10) is set equal to the settling time of the LMPC scheme. Problem (10) is solved with the commercial package DIDO, based on pseudospectral (PS) meth-

ods [15]. The total impulse (i.e., the fuel consumption) required by the maneuver and the CPU time needed by the solver (on a 2 Ghz, single-core CPU) are reported in Table 5 for the two approaches. It can be seen that the total impulse commanded by the LMPC scheme can be more than two times higher than the one provided by the fuel optimal, open-loop solution PS. According to Table 4, a significant part of this mismatch is due to measurement noise (which does not affect the PS solution), while the rest arises from the approximations made in the design of the LMPC scheme. On the other hand, the explicit LMPC solution is computed approximately 400 times faster than the PS solution.

Based on these data, and in view of the high specific impulse of the PPT technology, it can be concluded that the new design provides a reasonable trade-off between the performance and the computational burden of the control law.

Table 5. Open-loop (PS) and feedback solution (LMPC)

Type	Case 1 Impulse	Case 2 Impulse	CPU time
PS	0.09 Ns	0.11 Ns	~ 20 s
LMPC	0.16 Ns	0.25 Ns	~ 0.05 s
	+77%	+127%	1/400

VI. Conclusions

This paper has demonstrated that the use of Laguerre functions in combination with multi-parametric programming techniques can be effective in improving the computational efficiency of model predictive control when applied to the problem of low-thrust spacecraft rendezvous and proximity operations. The new design is general enough to systematically handle path constraints, as well as thrust magnitude and rate constraints. By use of the Laguerre parametrization of the input trajectories, an explicit controller featuring a long control horizon has been derived, thus circumventing the need of a dedicated solver on board the spacecraft. Simulation results show that the achievable performance, in terms of control accuracy and propellant usage, is suitable for autonomous rendezvous and docking between small three-axis stabilized spacecraft using electric propulsion. The applicability of the new design to more complex scenarios, such as circumnavigation and docking

with a tumbling target, still needs a deeper investigation.

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