

# System Identification

## Lab session #1 on Time Series Prediction

Consider the ARMA process,

$$y(t) - 1.8y(t-1) + 0.8075y(t-2) = e(t) + 0.5e(t-1)$$

where  $e(t)$  is a Gaussian white process with zero mean and variance equal to 1.

1. Compute the covariance function  $R_y(\tau)$  and plot it for  $|\tau| \leq 20$ .
2. Simulate a realization of process  $y(t)$ , for  $t = 1, \dots, N$ , with  $N = 100$ .
3. By using the realization obtained at point 2 and the Matlab function `xcov` (or `xcorr`), estimate the sample covariance

$$\hat{R}_y(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} (y(t+\tau) - \bar{y})(y(t) - \bar{y})$$

where  $\bar{y} = \frac{1}{N} \sum_{t=1}^N y(t)$  is the sample mean. Compare the obtained estimate with the true covariance computed at point 1. Repeat for longer realizations (e.g.,  $N = 100000$ ).

4. By using the Matlab function `predict`, compute the 1-step-ahead predictor  $\hat{y}(t+1|t)$  and the corresponding sample mean square error

$$\frac{1}{N} \sum_{t=1}^N \{y(t+1) - \hat{y}(t+1|t)\}^2.$$

Compare the true time series  $y(t)$  with the predicted one. Estimate the MSE from data and compare it with the theoretical one. Repeat for longer prediction horizons (e.g.,  $\hat{y}(t+3|t)$ ) and with realizations of different lengths. Observe the most significant differences.

5. Repeat the exercise for the ARMA process

$$y(t) - 1.8y(t-1) + 0.8075y(t-2) = e(t) + 2e(t-1)$$

with  $e(t)$  as above. Use the Matlab function `spectralfact` to compute the canonical spectral factor of  $y(t)$ . Compare the predictor and MSE with those obtained in item 4: what is the difference?