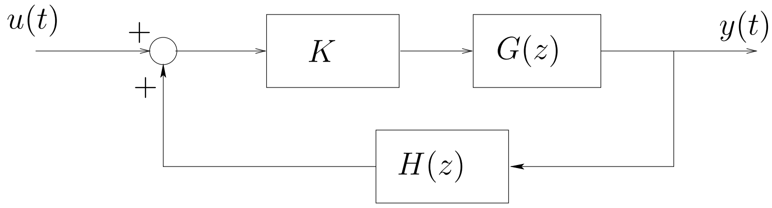
System Identification
Homework #1 on Time Series Prediction

Consider the stochastic process $y(t)$ generated as in following figure:



where

$$G(z) = \frac{z - \frac{3}{4}}{z - \frac{1}{2}} \ , \quad H(z) = \frac{1}{z} \ ,$$

K is a real constant and $u(t)$ is a Gaussian white process with zero mean and variance equal to 8.

1. Find the values of K for which $y(t)$ is asymptotically stationary.

   Hint: a second-order polynomial $P(x) = a_2x^2 + a_1x + a_0$ *has all the roots with modulus strictly less than 1 if and only if:* $P(1) > 0$, $P(-1) > 0$, $|a_2| > |a_0|$.

2. Find the spectrum $\Phi_y(z)$ of $y(t)$ and write the equation of $y(t)$ in the form of a MA, AR or ARMA process.

3. Set $K = 1$ and simulate a realization of process $y(t)$, for $t = 1, \ldots, N$, with $N = 1000$.

4. By using the realization obtained in item 3, compute the 2-step-ahead predictor $\hat{y}(t+2|t)$ and the corresponding sample mean square error. Compare the true time series $y(t)$ with the predicted one.

5. Compute the theoretical MSE of the Wiener predictor and compare it to the sample one computed in the previous item. Find a meaningful numerical procedure to improve the sample estimate of the MSE.