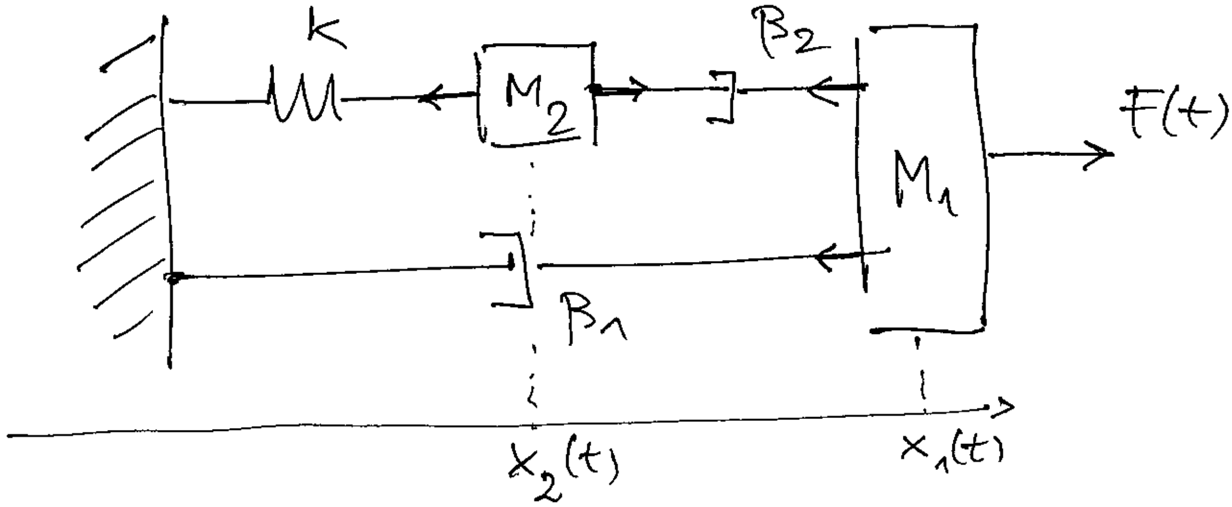
Elaborato 1 _ Esercizio 1



$$M_1 \ddot{x}_1(t) = F(t) - \beta_1 \dot{x}_1(t) - \beta_2 \left(\dot{x}_1(t) - \dot{x}_2(t) \right)$$

$$M_2 \ddot{x}_2(t) = \beta_2 \left(\dot{x}_1(t) - \dot{x}_2(t) \right) - k\, x_2(t)$$

$$z(t) = \begin{bmatrix} x_1(t) \\ \dot{x}_1(t) \\ x_2(t) \\ \dot{x}_2(t) \end{bmatrix}$$

$$\dot{z}_1(t) = z_2(t)$$

$$\dot{z}_2(t) = \frac{1}{M_1} F(t) - \frac{\beta_1}{M_1} z_2(t) - \frac{\beta_2}{M_1} \left(z_2(t) - z_4(t) \right)$$

$$\dot{z}_3(t) = z_4(t)$$

$$\dot{z}_4(t) = \frac{\beta_2}{M_2} \left(z_2(t) - z_4(t) \right) - \frac{k}{M_2} z_3(t)$$

$$y(t) = z_1(t)$$

Elaborato 2, Pb. 1

$x_1(k)$: under 15 nel quinquennio k
$x_2(k)$: over 15 " " "

$$x_1(k+1) = \alpha_2 x_2(k)$$

immigrati

$$x_2(k+1) = u(k) + (1-\mu_1)x_1(k) + (1-\mu_2)x_2(k)$$

$$y(k) = x_2(k)$$

$$A = \begin{bmatrix} 0 & \alpha_2 \\ 1-\mu_1 & 1-\mu_2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad D = [0]$$

$$G(z) = C(zI-A)^{-1}B + D = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z & -\alpha_2 \\ -(1-\mu_1) & z-(1-\mu_2) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{z}{z^2 - (1-\mu_2)z - \alpha_2(1-\mu_1)} = \frac{Y(z)}{U(z)}$$

$$y(k+2) - (1-\mu_2)y(k+1) - \alpha_2(1-\mu_1)y(k) = u(k+1) \qquad \text{i/o}$$

$$\mu_1 = 0.1, \ \mu_2 = 0.2 \implies A = \begin{bmatrix} 0 & \alpha_2 \\ 0.9 & 0.8 \end{bmatrix}$$

$$\det(\lambda I - A) = \det\begin{pmatrix} \lambda & -\alpha_2 \\ -0.9 & \lambda-0.8 \end{pmatrix} = \lambda^2 - 0.8\lambda - 0.9\alpha_2$$

$$\lambda = 0.4 \pm \sqrt{0.16+0.9\alpha_2}$$

$$\Rightarrow 0.4 + \sqrt{0.16+0.9\alpha_2} \geq 1$$

$$\sqrt{0.16+0.9\alpha_2} \geq 0.6$$

$$0.16+0.9\alpha_2 \geq 0.36$$

$$0.9\alpha_2 \geq 0.2$$

$$\boxed{\alpha_2 \geq \frac{2}{9}}$$

Elaborato 3 - Es. 3

1) $$W(s) = \frac{1}{s+1} \cdot \left[\frac{\frac{1}{s+3}}{1 + \frac{1}{s+3} \cdot \frac{s+2}{s+1}} \right] =$$

$$= \frac{1}{\cancel{s+1}} \cdot \frac{\cancel{s+1}}{(s+3)(s+1) + s+2} =$$

$$= \frac{1}{s^2+3s+s+3+s+2} = \frac{1}{s^2+5s+5} = \frac{Y(s)}{U(s)}$$

$$(s^2+5s+5)Y(s) = U(s)$$

$$\ddot{y}(t) + 5\dot{y}(t) + 5y(t) = u(t)$$

2) $$\dot{x}(t) = Ax(t) + Bu(t)$$

$$X_1(s) = \frac{1}{s+1} \cdot U(s)$$

$$(s+1)X_1(s) = U(s) \xrightarrow{\mathcal{L}^{-1}} \dot{x}_1(t) + x_1(t) = u(t)$$

$$\rightarrow \dot{x}_1(t) = -x_1(t) + u(t)$$

$$X_2(s) = \frac{1}{s+3}\left[X_1(s) - X_3(s)\right]$$

$$\dot{x}_2(t) + 3x_2(t) = x_1(t) - x_3(t)$$

$$\rightarrow \dot{x}_2(t) = x_1(t) - 3x_2(t) - x_3(t)$$

$$X_3(s) = \frac{s+2}{s+1} X_2(s)$$

$$(s+1) X_3(s) = (s+2) X_2(s)$$

$$\dot{x}_3(t) + x_3(t) = \dot{x}_2(t) + 2x_2(t)$$

$$x_1(t) - 3x_2(t) - x_3(t)$$

$$\rightarrow \dot{x}_3(t) = -x_3(t) + \underline{x_1(t)} - \underline{\underline{3x_2}}(t) - x_3(t) + 2\underline{\underline{x_2}}(t)$$

$$= x_1(t) - x_2(t) - 2x_3(t)$$

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -3 & -1 \\ 1 & -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \end{bmatrix} u(t)$$

3) $\lim_{t \to +\infty} y_f(t) = ?$ $\qquad u(t) = 1(t)$

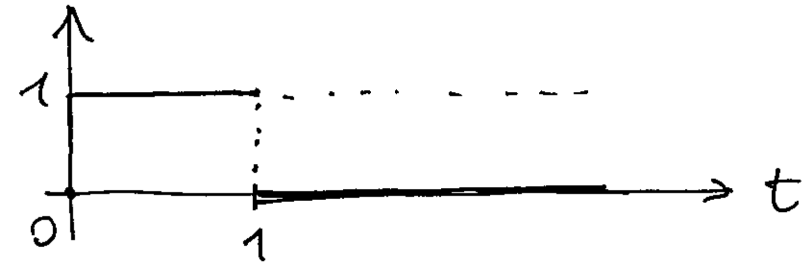
$$Y_f(s) = \frac{1}{s^2 + 5s + 5} \cdot \frac{1}{s}$$

poli: $-5 \pm \sqrt{25-20} = -5 \pm \sqrt{5} < 0$

$$\lim_{t \to +\infty} y_f(t) = \lim_{s \to 0} s \cdot Y_f(s) = \lim_{s \to 0} \cancel{s} \cdot \overset{W}{\cancel{G}}(s) \cdot \frac{1}{\cancel{s}} =$$

$$= W(0) = \frac{1}{5}$$

4)



$$u(t) = \mathbb{1}(t) - \mathbb{1}(t-1) \longrightarrow U(s) = \frac{1}{s} - \frac{1}{s} e^{-s\cdot 1}$$

$$Y_f(s) = G(s)\cdot U(s) = \frac{1}{s^2+5s+5} \cdot \left[\frac{1}{s} - \frac{1}{s}\cdot e^{-s}\right]$$

$$q(t) = \mathcal{L}^{-1}\left[G(s)\cdot\frac{1}{s}\right] \Rightarrow y_f(t) = q(t)\cdot\mathbb{1}(t) - q(t-1)\cdot\mathbb{1}(t-$$

$$q(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2+5s+5}\cdot\frac{1}{s}\right] =$$

$$= \mathcal{L}^{-1}\left[\frac{R_1}{s} + \frac{R_2}{s+5-\sqrt{5}} + \frac{R_3}{s+5+\sqrt{5}}\right]$$

$$R_1 = \frac{1}{5} = W(0)$$

$$R_2 = \lim_{s\to -5+\sqrt{5}} \left[s-(-5+\sqrt{5})\right]\frac{1}{s^2+5s+5}\cdot\frac{1}{s} =$$

$$= \lim_{s\to -5+\sqrt{5}} \frac{1}{s-(-5-\sqrt{5})}\cdot\frac{1}{s} = \frac{1}{2\sqrt{5}}\cdot\frac{1}{-5+\sqrt{5}} =$$

$$= \frac{1}{10-10\sqrt{5}}$$

$$R_3 = \lim_{s\to -5-\sqrt{5}} \frac{1}{s-(-5+\sqrt{5})}\cdot\frac{1}{s} = \frac{1}{-2\sqrt{5}}\cdot\frac{1}{-5-\sqrt{5}} =$$

$$= \frac{1}{10+10\sqrt{5}}$$

$$q(t) = \left(\frac{1}{5} + \frac{1}{10-10\sqrt{5}} e^{(-5+\sqrt{5})t} + \frac{1}{10+10\sqrt{5}} e^{(-5-\sqrt{5})t}\right) \mathbb{1}($$

Elaborato 4. Problema 4

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & \frac{1}{4} \\ 2 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \qquad D = [0]$$

$$G(z) = [0 \quad 1 \quad 1] \begin{bmatrix} z & 0 & 0 \\ -1 & z & -\frac{1}{4} \\ -2 & -1 & z \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} =$$

$$= \frac{[0 \quad 1 \quad 1] \begin{bmatrix} * \\ z+\frac{1}{2} \\ 1+2z \end{bmatrix}}{z\left(z^2-\frac{1}{4}\right)} =$$

$$= \frac{z+\frac{1}{2}+1+2z}{z\left(z^2-\frac{1}{4}\right)} = \frac{3z+\frac{3}{2}}{z\left(z^2-\frac{1}{4}\right)} =$$

$$= \frac{3\cancel{\left(z+\frac{1}{2}\right)}}{z\left(z-\frac{1}{2}\right)\cancel{\left(z+\frac{1}{2}\right)}} = \frac{3}{z\left(z-\frac{1}{2}\right)}$$

3) Determinare $u(k)$ in modo che $y_f(k) = (k-1) \cdot 1\!\!1(k-1)$

$$Y_f(z) = G(z) \cdot U(z)$$

$$\mathcal{Z}[k \cdot 1\!\!1(k)] = \frac{z}{(z-1)^2} \Rightarrow \mathcal{Z}[(k-1) \cdot 1\!\!1(k-1)] =$$

$$= z^{-1} \cdot \frac{z}{(z-1)^2} = \frac{1}{(z-1)^2}$$

$$U(z) = \frac{Y_f(z)}{G(z)} = \frac{1}{(z-1)^2} \cdot \frac{z\left(z-\frac{1}{2}\right)}{3}$$

$$\frac{U(z)}{z} = \frac{\left(z-\frac{1}{2}\right)\frac{1}{3}}{(z-1)^2} = \frac{A}{z-1} + \frac{B}{(z-1)^2}$$

$$A(z-1)+B = \frac{1}{3}z - \frac{1}{6}$$

$$A = \frac{1}{3}$$

$$-A+B = -\frac{1}{6}$$

$$B = \frac{1}{6}$$

$$u(k) = \frac{1}{3}\cdot \mathbb{1}(k) + \frac{1}{6}k\cdot \mathbb{1}(k)$$

Esercizio 2 _ Esercitazione 1

1) $$J \cdot \ddot{\theta}(t) = \sum_i \tau_i(t) =$$
$$= \tau(t) - K \cdot \theta(t) - \beta \dot{\theta}(t)$$

$$J \ddot{\theta}(t) + \beta \dot{\theta}(t) + K \theta(t) = \tau(t)$$

$u = \tau$
$y = \theta$

$$G(s) = \frac{1}{Js^2 + \beta s + K} = \frac{1}{2s^2 + \beta s + 3}.$$

2) $\beta > 0 \Rightarrow$ poli a parte reale negativa $\Rightarrow$
$\Rightarrow$ modi convergenti

$\beta = 0 \Rightarrow$ poli a parte reale nulla $\Rightarrow$
$\Rightarrow$ modi limitati non convergenti

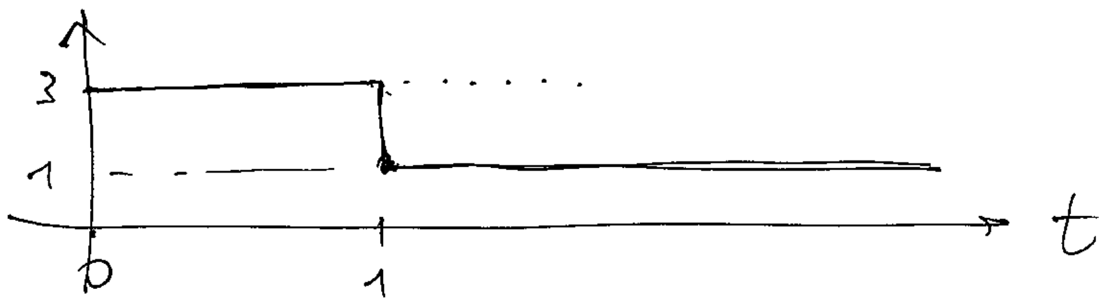
3) poli: $\dfrac{-\beta \pm \sqrt{\beta^2 - 24}}{4}$

modi: $e^{\alpha t} \cos(\omega t + \phi)$, $\alpha < -1$

$$\Rightarrow \begin{cases} \beta^2 - 24 < 0 & \text{(modi pseudoperiodici)} \\ -\dfrac{\beta}{4} < -1 & (\mathrm{Re}[\text{poli}] < -1) \end{cases}$$

$$\Rightarrow \underline{4 < \beta < \sqrt{24}}$$

4) $\beta = 5$ $\quad u(t) = \begin{cases} 3 & 0 < t \leq 1 \\ 1 & t > 1 \end{cases}$



$$u(t) = 3 \cdot \mathbb{1}(t) - 2 \cdot \mathbb{1}(t-1)$$

$$U(s) = 3 \cdot \frac{1}{s} - 2 \cdot \frac{1}{s} \cdot e^{-s \cdot 1}$$

$$Y_f(s) = G(s) \cdot U(s) = \frac{1}{2s^2+5s+3} \cdot \left\{ 3 \cdot \frac{1}{s} - 2 \cdot \frac{1}{s} e^{-s} \right\}$$

se $h(t) = \mathcal{L}^{-1}\left[\frac{1}{2s^2+5s+3} \cdot \frac{1}{s} \right]$

allora $y_f(t) = 3 \cdot h(t) \, \mathbb{1}(t) - 2\, h(t-1) \cdot \mathbb{1}(t-1)$

Esercizio 1 25/11/2013

1) $x_1(k+1) = 0.3\, x_2(k) + 0.6\, x_1(k)$

$x_2(k+1) = 0.4\, x_1(k) + 0.7\, x_2(k)$

2) $x(k+1) = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} x(k)$

$x(\emptyset) = \begin{pmatrix} N/2 \\ N/2 \end{pmatrix}$ N: numero totale di auto

$\lim_{k \to +\infty} x_\ell(k) = ?$

$\det(zI - A) = \det \begin{bmatrix} z-0.6 & -0.3 \\ -0.4 & z-0.7 \end{bmatrix} =$

$= (z-0.6)(z-0.7) - 0.12 = z^2 - 1{,}3z + 0.3 =$

$= (z-1)(z-0.3)$

$\lim_{k \to +\infty} x_\ell(k) = \lim_{z \to 1} (z-1) \cdot X_\ell(z) =$

$= \lim_{z \to 1} (z-1)(zI-A)^{-1} z \cdot x(\emptyset) =$

$= \lim_{z \to 1} \cancel{(z-1)} \dfrac{\begin{bmatrix} z-0.7 & 0.3 \\ 0.4 & z-0.6 \end{bmatrix}}{\cancel{(z-1)}(z-0.3)} \cdot z\, x(\emptyset) =$

$= \dfrac{\begin{bmatrix} 0.3 & 0.3 \\ 0.4 & 0.4 \end{bmatrix}}{0.7} \cdot x(\emptyset) = \begin{bmatrix} \frac{3}{7} & \frac{3}{7} \\ \frac{4}{7} & \frac{4}{7} \end{bmatrix} x(\emptyset) =$

$= \begin{pmatrix} 3/7 \\ 4/7 \end{pmatrix} \quad \forall\, x(\emptyset) : \; x_1(\emptyset) + x_2(\emptyset) = 1$

3) $y(k) = x_1(k)$ $\qquad x(\emptyset) = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$

$y_\ell(k) = ?$

$$Y_\ell(z) = C(zI-A)^{-1} z \cdot x(\emptyset) =$$

$$= [1 \quad 0] \frac{\begin{pmatrix} z-0.7 & 0.3 \\ 0.4 & z-0.6 \end{pmatrix} z}{(z-1)(z-0.3)} \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} =$$

$$= \frac{0.5(z-0.7) + 0.15}{(z-1)(z-0.3)} \cdot z =$$

$$= \frac{(0.5z - 0.2)z}{(z-1)(z-0.3)}$$

$$\frac{Y_\ell(z)}{z} = \frac{R_1}{z-1} + \frac{R_2}{z-0.3}$$

$$R_1(z-0.3) + R_2(z-1) = 0.5z - 0.2$$

$$\begin{cases} R_1 + R_2 = 0.5 \\ -0.3R_1 - R_2 = -0.2 \end{cases} \qquad 0.7 R_1 = 0.3 \qquad R_1 = \frac{3}{7}$$

$$R_2 = 0.5 - \frac{3}{7} = \frac{1}{2} - \frac{3}{7} = \frac{1}{14}$$

$$y_\ell(k) = \frac{3}{7} \cdot \mathbb{1}(k) + \frac{1}{14}(0.3)^k \cdot \mathbb{1}(k)$$

Esercizio 3   25/11/2013

1) $$W(z) = \frac{G_1(z)\,G_2(z)}{1 + G_1(z)\,G_2(z)\,\alpha} =$$

$$= \frac{\frac{1}{z-\frac{1}{4}} \cdot \frac{z}{z-1}}{1 + \frac{1}{z-\frac{1}{4}} \cdot \frac{z}{z-1} \cdot \frac{5}{4}} =$$

$$= \frac{z}{(z-1)(z-\frac{1}{4}) + \frac{5}{4}z} =$$

$$= \frac{z}{z^2 - \cancel{\frac{5}{4}z} + \frac{1}{4} + \cancel{\frac{5}{4}z}} = \frac{z}{z^2 + \frac{1}{4}}$$

2) Modi : ?

poli $\pm \frac{1}{2} j \implies$ modi : $\left(\frac{1}{2}\right)^k e^{\pm j \frac{\pi}{2} k}$

ovvero $\left(\frac{1}{2}\right)^k \cos\left(\frac{\pi}{2}k\right)$ $\left(\frac{1}{2}\right)^k \sin\left(\frac{\pi}{2}k\right)$

3) Determinare $r(k)$ in modo che $y_f(k) = 1(k-1)$

cioè $Y_f(z) = \frac{1}{z-1} = \cancel{z}^{-1} \cdot \frac{\cancel{z}}{z-1}$

$$\frac{1}{z-1} = \frac{z}{z^2+\frac{1}{4}} \cdot R(z)$$

$$R(z) = \frac{z^2+\frac{1}{4}}{z(z-1)} = 1 + \frac{z+\frac{1}{4}}{z(z-1)} =$$

$$= 1 + \frac{R_1}{z} + \frac{R_2}{z-1} = 1 - \frac{1/4}{z} + \frac{5/4}{z-1}$$

$$\left.\begin{array}{l} R_1 + R_2 = 1 \\ -R_1 = \frac{1}{4} \end{array}\right\}$$

$$r(k) = \mathcal{Z}^{-1}[R(z)] = \delta(k) - \frac{1}{4}\delta(k-1) + \frac{5}{4}\mathbb{1}(k-1)$$

Esercizio 2 25/11/2013

I) $\dot{x}_1 = \dot{d} = x_2$ $\qquad$ $y_1 = d = x_1$

$\dot{x}_2 = \ddot{d} = 0$ $\qquad$ $y_2 = h = x_3$

$\dot{x}_3 = \dot{h} = x_4$

$\dot{x}_4 = \ddot{h} = u$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

II) $G(s)$ da $u(t)$ a $y_2(t)$

$$G(s) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & -1 \\ 0 & 0 & \boxed{0} & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} =$$

$$= \frac{\mathrm{Agg}_{4,3}}{s^4} = \frac{(-1)^{4+3} \cdot \det\begin{pmatrix} s & -1 & 0 \\ 0 & s & 0 \\ 0 & 0 & -1 \end{pmatrix}}{s^4} = \frac{s^2}{s^4} = \frac{1}{s^2}$$

Es 3 20/11/2014

$$\ddot{y}(t) + 2\dot{y}(t) + (5-k)y(t) = u(t)$$

1) $K \in \mathbb{R}$: modi convergenti

$$G(s) = \frac{1}{s^2 + 2s + 5 - K}$$

Poli a parte reale negativa $\Rightarrow 5-k>0 \Rightarrow k<5$

$$\Delta = 4 - 4(5-k) = 4K - 16$$

Modi aperiodici per $k \in [4, 5)$

Modi pseudoperiodici per $k<4$

2) k : $u(t) = \cos(t) \Rightarrow y_{perm} = M \sin(t)$

$$y_{perm}(t) = |G(j)| \cos(t + \angle G(j)) = M \cos\left(t - \frac{\pi}{2}\right)$$

$$\angle G(j) = \angle \frac{1}{-1+2j+5-k} = \angle \frac{1}{4-k+2j} = -\frac{\pi}{2}$$

$$-\angle 4-k+2j = -\frac{\pi}{2}$$

$$\arctan \frac{2}{4-k} = \frac{\pi}{2} \Rightarrow \underline{\underline{k=4}}$$

$$M = |G(j)| = \left| \frac{1}{-1+2j+5-4} \right| = \left| \frac{1}{2j} \right| = \frac{1}{2}$$

$\underline{\underline{K=4}}$ $\quad y_{perm}(t) = \frac{1}{2} \cos\left(t - \frac{\pi}{2}\right) = \frac{1}{2} \sin(t)$