1.1 $\dot{x}_1 = -2x_1 + r - w = -2x_1 + r - x_2$

$\dot{x}_2 = \beta x_2 + x_1$

$y = x_1$

$A = \begin{bmatrix} -2 & -1 \\ 1 & \beta \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = [1 \quad 0] \quad D = [0]$

1.2 $\det(\lambda I - A) = (\lambda+2)(\lambda-\beta)+1 = \lambda^2 + (2-\beta)\lambda + 1 - 2\beta$

$\begin{cases} \beta < 2 \\ \beta < \frac{1}{2} \end{cases} \Rightarrow$

$\beta < \frac{1}{2}$ sistema asintoticamente stabile

$\beta = \frac{1}{2}$ marginalmente stabile

$\beta > \frac{1}{2}$ instabile

1.3 $G(s) = \dfrac{s}{s^2+2s+1}$

$\lim_{t\to+\infty} y_f(t) = G(0) = 0$

1.4. $|G(j\omega)| > \frac{1}{4} \Rightarrow \dfrac{\omega}{1+\omega^2} > \dfrac{1}{4}$

$\omega^2 - 4\omega + 1 < 0 \qquad \omega_{1,2} = 2 \pm \sqrt{3}$

$\Rightarrow \omega \in (2-\sqrt{3},\ 2+\sqrt{3})$

2.1 $g(k) = (-1)^k \cdot \mathbb{1}(k) \Rightarrow G(z) = \dfrac{z}{z+1}$

Rappresentazione i/o: $y(k+1) + y(k) = u(k+1)$

2.2 $Y_f(z) = \dfrac{z}{z+1} \cdot \dfrac{z}{z-1} = \dfrac{1}{2}\left\{\dfrac{z}{z+1} + \dfrac{z}{z-1}\right\}$

$y_f(k) = \left\{\dfrac{1}{2} + \dfrac{1}{2}(-1)^k\right\} \cdot \mathbb{1}(k)$

2.3 $Y_f(z) = c \cdot \frac{z}{z-1} = \frac{z}{z+1} \cdot U(z) \Rightarrow U(z) = c \cdot \frac{z+1}{z-1}$

$u(k) = c\{\cdot 1\!\!1(k) + 1\!\!1(k-1)\}$.

2.4 $y(k) = -y(k-1) + u(k) = -y(k-1) + K y(k-1) + r(k)$

$\frac{Y(z)}{R(z)} = \frac{z}{z-(K-1)}$ $\qquad -1 < K-1 < 1 \Rightarrow K \in (0,2)$

3.1 $A = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ $\quad \det(\lambda I - A) = \left(\lambda - \frac{2}{3}\right)\left(\lambda - \frac{1}{2}\right) - \frac{1}{6} =$

$= \lambda^2 - \frac{7}{6}\lambda + \frac{1}{6} = (\lambda-1)\left(\lambda - \frac{1}{6}\right)$

modi: $1\!\!1(k)$ (limitato non convergente); $\left(\frac{1}{6}\right)^k 1\!\!1(k)$ (convergente)

Sistema marginalmente stabile.

3.2 $A - LC = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} - l_1 & \frac{1}{3} \\ \frac{1}{2} - l_2 & \frac{1}{2} \end{bmatrix}$

$\det(\lambda I - A + LC) = \left(\lambda - \frac{2}{3} + l_1\right)\left(\lambda - \frac{1}{2}\right) - \frac{1}{6} + \frac{1}{3} l_2 =$

$= \lambda^2 + \left(l_1 - \frac{7}{6}\right)\lambda + \frac{1}{6} - \frac{1}{2} l_1 + \frac{1}{3} l_2 = \lambda^2$

$\Rightarrow \begin{cases} l_1 - \frac{7}{6} = 0 \\ \frac{1}{6} - \frac{1}{2} l_1 + \frac{1}{3} l_2 = 0 \end{cases}$ $\qquad L = \begin{bmatrix} \frac{7}{6} \\ \frac{5}{4} \end{bmatrix}$

3.3 $\begin{cases} \bar{x}_1 = \frac{2}{3}\bar{x}_1 + \frac{1}{3}\bar{x}_2 \\ \bar{x}_2 = \frac{1}{2}\bar{x}_1 + \frac{1}{2}\bar{x}_2 + \frac{1}{16} - \bar{x}_1^2 \end{cases} \Rightarrow \begin{cases} \bar{x}_1 = \bar{x}_2 \\ \bar{x}_1^2 = \frac{1}{16} \end{cases} \Rightarrow \bar{x}_A = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}$ $\quad \bar{x}_B = \begin{pmatrix} -\frac{1}{4} \\ -\frac{1}{4} \end{pmatrix}$

3.4 $\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} - 2x_1 & \frac{1}{2} \end{bmatrix} = J$

$\det(\lambda I - J_{|\bar{x}_A}) = \left(\lambda - \frac{2}{3}\right)\left(\lambda - \frac{1}{2}\right) \Rightarrow \bar{x}_A$ asintoticamente stabile

$\det(\lambda I - J_{|\bar{x}_B}) = \left(\lambda - \frac{2}{3}\right)\left(\lambda - \frac{1}{2}\right) - \frac{1}{3} = \lambda^2 - \frac{7}{6}\lambda = \lambda\left(\lambda - \frac{7}{6}\right)$

$\Rightarrow \bar{x}_B$ instabile