1.1 $$W(s) = \frac{C(s)\,G(s)}{1+C(s)\,G(s)} = \frac{K}{s^3+5s^2+6s+K}$$

$$\begin{array}{ccc} 3 & 1 & 6 \\ 2 & 5 & K \\ 1 & \frac{30-K}{5} & \\ 0 & K & \end{array} \quad \Rightarrow \quad K \in (0,30)$$

1.2 $$y_{imp}(t) = \mathcal{L}^{-1}\left\{ \frac{8}{s^3+5s^2+6s+8} \right\} = \mathcal{L}^{-1}\left\{ \frac{8}{(s+4)(s^2+s+2)} \right\}$$

$$\frac{8}{(s+4)\left(\left(s+\frac{1}{2}\right)^2+\frac{7}{4}\right)} = \frac{A}{s+4} + \frac{B\left(s+\frac{1}{2}\right)}{s^2+s+2} + \frac{C\frac{\sqrt{7}}{2}}{s^2+s+2}$$

$$A(s^2+s+2) + B\left(s+\tfrac{1}{2}\right)(s+4) + C\frac{\sqrt{7}}{2}(s+4) = 8$$

$$\begin{cases} A+B=0 \\ A+\frac{9}{2}B+\frac{\sqrt{7}}{2}C=0 \\ 2A+2B+2\sqrt{7}\,C=8 \end{cases} \qquad \begin{array}{l} A=\frac{4}{7} \\ B=-\frac{4}{7} \\ C=\frac{4}{\sqrt{7}} \end{array}$$

$$y_{imp}(t) = \left\{ \frac{4}{7}e^{-4t} - \frac{4}{7}e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{7}}{2}t\right) + \frac{4}{\sqrt{7}}e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{7}}{2}t\right) \right\} \cdot 1(t)$$

1.3 $$Q(s) = \frac{G(s)}{1+C(s)G(s)} = \frac{s+3}{s^3+5s^2+6s+K}$$

Per $K \in (0,30)$

$$\lim_{t\to+\infty} y_f(t) = \lim_{s\to 0} s\cdot Q(s)\cdot\frac{1}{s} = \frac{3}{K} < \frac{1}{5}$$

$$\Rightarrow \quad K \in (15,30)$$

1.4 $$Q(j\sqrt{2}) = \frac{3+j\sqrt{2}}{(4+j\sqrt{2})(j\sqrt{2})} = \frac{3+j\sqrt{2}}{-2+j4\sqrt{2}} = \frac{1}{18} - \frac{7\sqrt{2}}{18}j$$

$$|Q(j\sqrt{2})| = \frac{\sqrt{11}}{6} \qquad \measuredangle Q(j\sqrt{2}) = -\arctan(7\sqrt{2})$$

$$y_{perm}(t) = \frac{\sqrt{11}}{6}\cos\left(\sqrt{2}\,t - \arctan(7\sqrt{2})\right)$$

2.1 $Y_f(z) = \left[C(zI-A)^{-1}B + D\right] \cdot U_1(z) =$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z & 0 & 0 \\ -1 & z & 0 \\ -1 & -1 & z \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \frac{z}{z-1} =$$

$$= \frac{z+1}{z^3} \cdot \frac{z}{z-1} = \frac{1}{z^2} \left\{ \frac{z}{z-1} + \frac{1}{z-1} \right\}$$

$$y_f(k) = \mathbb{1}(k-2) + \mathbb{1}(k-3)$$

2.2 $A = \begin{bmatrix} 0 & 0 & \alpha \\ 1 & 0 & \alpha \\ 1 & 1 & 0 \end{bmatrix}$ $\qquad I - A = \begin{bmatrix} 1 & 0 & -\alpha \\ -1 & 1 & -\alpha \\ -1 & -1 & 1 \end{bmatrix}$

$$\det(I-A) = 1 - \alpha - 2\alpha = 1 - 3\alpha = 0 \qquad \alpha = \frac{1}{3}$$

$$\ker \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ -1 & 1 & -\frac{1}{3} \\ -1 & -1 & 1 \end{bmatrix} = L \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\det \begin{bmatrix} \lambda & 0 & -\frac{1}{3} \\ -1 & \lambda & -\frac{1}{3} \\ -1 & -1 & \lambda \end{bmatrix} = \lambda\left(\lambda^2 - \frac{1}{3}\right) - \frac{1}{3}(\lambda+1) =$$

$$= \lambda^3 - \frac{2}{3}\lambda - \frac{1}{3} = (\lambda - 1)\left(\lambda^2 + \lambda + \frac{1}{3}\right)$$

$\lambda_1 = 1 \qquad \lambda_{2,3} = -\frac{1}{2} \pm j\frac{1}{2\sqrt{3}}$ sistema marginalmente stabile

2.3 a) $R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ sistema completamente raggiungibile (in 2 passi)

b) $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ sistema completamente raggiungibile

c) $\mathcal{R} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ Sistema non completamente raggiungibile

$$\chi^R = L\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

2.4 $\begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} u_1(1) \\ u_2(1) \\ u_1(0) \\ u_2(0) \end{pmatrix}$

$$\begin{cases} u_1(0) + u_2(0) = 10 \\ u_2(1) + u_1(0) = 10 \\ u_1(1) = 10 \end{cases}$$

3.1 $Y_\ell(s) = C(sI - A)x(0) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ 4 & s+5 \end{bmatrix}^{-1} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} =$

$$= \frac{(s+5)\,x_1(0) + x_2(0)}{s^2 + 5s + 4} = \frac{(s+5)x_1(0) + x_2(0)}{(s+1)(s+4)}$$

$s = -1 \implies 4x_1(0) + x_2(0) = 0$

3.2 $A + BF = \begin{bmatrix} 0 & 1 \\ -4+f_1 & -5+f_2 \end{bmatrix}$

$\det(\lambda I - A - BF) = \lambda^2 + 9$

$\lambda^2 + (5 - f_2)\lambda + 4 - f_1 = \lambda^2 + 9$ $\qquad F = \begin{bmatrix} -5 & 5 \end{bmatrix}$

3.3 $\begin{cases} x_2 = 0 \\ -4x_1 - 5x_2 + u = 0 \end{cases}$

$x_2 = 0 \qquad u = 4x_1$

$$8x_1 = 4x_1 \quad \Rightarrow \quad x_1 = 0 < \beta \qquad \text{OK}$$

$$8\sqrt{\beta x_1} = 4x_1 \quad \Rightarrow \quad x_1 = 4\beta > \beta \qquad \text{OK}$$

$$\bar{x}_A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \bar{x}_B = \begin{pmatrix} 4\beta \\ 0 \end{pmatrix}$$

3,4 $$J = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ -4 + \frac{\partial u}{\partial x_1} & -5 \end{bmatrix}$$

$$J\Big|_{x=\bar{x}_A} = \begin{bmatrix} 0 & 1 \\ -4+8 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & -5 \end{bmatrix}$$

$$\det(\lambda I - J) = \lambda^2 + 5\lambda - 4 \quad \Rightarrow \quad \bar{x}_A \ \text{INSTABILE}$$

$$J\Big|_{x=\bar{x}_B} = \begin{bmatrix} 0 & 1 \\ -4 + \frac{4\beta}{\sqrt{\beta x_1}} & -5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -5 \end{bmatrix}$$

$$\det(\lambda I - J) = \lambda^2 + 5\lambda + 2 \quad \Rightarrow \quad \bar{x}_B \ \text{ASINTOTICAMENTE STABILE}$$