1.1 $x_1(t) = y(t) \quad x_2(t) = \dot{y}(t)$

$$M\ddot{y}(t) = u(t) - Ky(t)$$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\frac{K}{M} x_1(t) + \frac{1}{M} u(t) = -4x_1(t) + u(t) \\ y(t) = x_1(t) \end{cases}$$

1.2 $A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [1 \quad 0] \quad D = \emptyset$

$\det(\lambda I - A) = \lambda^2 + 4 = 0 \qquad \lambda = \pm 2j$

Modi: $\cos(2t)$, $\sin(2t)$

Sistema marginalmente stabile.

$G(s) = C(sI-A)^{-1}B = \frac{1}{s^2+4}$ Non è stabile in senso ILUL.

1.3 $Y_\ell(s) = C(sI-A)^{-1}x(0) = [1 \quad 0] \begin{bmatrix} s & -1 \\ 4 & s \end{bmatrix}^{-1} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} =$

$$= \frac{s \cdot x_1(0) + x_2(0)}{s^2+4} = 5 \cdot \frac{2}{s^2+4}$$

$x_1(0) = \emptyset \, , \quad x_2(0) = 10$

1.4 $u(t) = 4 \cdot 1\!\!1(t)$

$$Y_f(s) = \frac{1}{s^2+4} \cdot \frac{4}{s} = \frac{1}{s} - \frac{s}{s^2+4}$$

$y_f(t) = 1 - \cos(2t)$

$\Rightarrow y_{max} = 2 \text{ m}$

2.1 $A = \begin{bmatrix} \frac{1}{2} & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ \alpha \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$

Autovalori di $A = \left\{ \frac{1}{2}, 1, 0 \right\}$

Modi: $\begin{cases} \left(\frac{1}{2}\right)^k & \text{convergente} \\ \mathbb{1}(k) & \text{limitato non convergente} \\ \delta(k) & \text{convergente (impulsivo).} \end{cases}$

2.2 $Y_{imp}(z) = G(z) = C(zI - A)^{-1} B + D =$

$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z-\frac{1}{2} & -1 & -1 \\ 0 & z-1 & 0 \\ 0 & -2 & z \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} =$

$= \dfrac{z+2}{z(z-1)(z-\frac{1}{2})} = \dfrac{4}{z} + \dfrac{6}{z-1} - \dfrac{10}{z-\frac{1}{2}}$

$y_{imp}(k) = 4\,\delta(k-1) + \left\{ 6 - 10\left(\frac{1}{2}\right)^{k-1} \right\} \cdot \mathbb{1}(k-1)$

2.3 $R = \begin{bmatrix} 0 & 1+\alpha & \frac{7+\alpha}{2} \\ 1 & 1 & 1 \\ \alpha & 2 & 2 \end{bmatrix}$

$\det R = \alpha\left(1+\alpha - \frac{7+\alpha}{2}\right) - (2+2\alpha-7-\alpha) =$

$= \left(\frac{\alpha}{2} - 1\right)(\alpha - 5)$

Sistema completamente raggiungibile per $\alpha \neq 2, \quad \alpha \neq 5$

2.4

$$A+BF = \begin{bmatrix} \frac{1}{2} & 1 & 1 \\ f_1 & 1+f_2 & f_3 \\ 0 & 2 & 0 \end{bmatrix}$$

$$\det(\lambda I - A - BF) = \lambda^3$$

$$\begin{vmatrix} \lambda-\frac{1}{2} & -1 & -1 \\ -f_1 & \lambda-1-f_2 & -f_3 \\ 0 & -2 & \lambda \end{vmatrix} = \left(\lambda-\frac{1}{2}\right)\left(\lambda^2-(1+f_2)\lambda-2f_3\right)+f_1(-\lambda-2)=$$

$$= \lambda^3+\lambda^2\left(-\frac{1}{2}-1-f_2\right)+\lambda\left(-2f_3+\frac{1}{2}+\frac{1}{2}f_2-f_1\right)-2f_1+f_3$$

$$\begin{cases} -\frac{3}{2}-f_2=0 \\ \frac{1}{2}-f_1+\frac{1}{2}f_2-2f_3=0 \\ -2f_1+f_3=0 \end{cases} \qquad \begin{aligned} & f_2=-\frac{3}{2} \\ & f_1+2f_3=-\frac{1}{4} \\ & f_3=2f_1 \end{aligned} \qquad \begin{aligned} & f_1=-\frac{1}{20} \\ & f_3=-\frac{1}{10} \end{aligned}$$

$$F=\begin{bmatrix} -\frac{1}{20} & -\frac{3}{2} & -\frac{1}{10} \end{bmatrix}$$

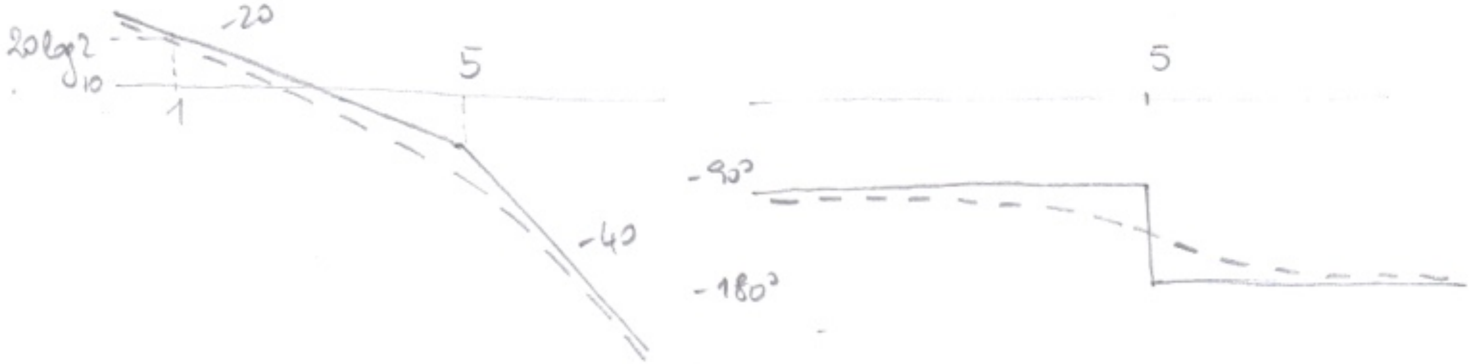
3.1

$$\mathring{x}_1(t)=u(t)$$
$$\mathring{x}_2(t)+5x_2(t)=10\,x_1(t)$$
$$y(t)=x_2(t)$$

$$A=\begin{bmatrix} 0 & 0 \\ 10 & -5 \end{bmatrix} \quad B=\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C=\begin{bmatrix} 0 & 1 \end{bmatrix} \quad D=0$$

3.2

$$W(s)=\frac{10}{s(s+5)}=\frac{2}{s\left(1+\frac{1}{5}s\right)}$$



3.3 $\dot{x}_1(t) = 1 - x_2^2(t)$

$\dot{x}_2(t) = 10\, x_1(t) - 5\, x_2(t)$

$$\begin{cases} x_2^2 = 1 \\ x_1 = \frac{1}{2} x_2 \end{cases} \qquad A = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} \quad B = \begin{pmatrix} -\frac{1}{2} \\ -1 \end{pmatrix}$$

3.4 $$J = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & -2x_2 \\ 10 & -5 \end{bmatrix}$$

$$J|_A = \begin{bmatrix} 0 & -2 \\ 10 & -5 \end{bmatrix} \qquad \det(\lambda I - J|_A) = \lambda^2 + 5\lambda + 20$$

A → ASINTOTICAMENTE STABILE

$$J|_B = \begin{bmatrix} 0 & 2 \\ 10 & -5 \end{bmatrix} \qquad \det(\lambda I - J|_B) = \lambda^2 + 5\lambda - 20$$

B → INSTABILE