SOLUZIONI COMPITO SISTEMI DINAMICI 25/1/2021 1

1.1)
$$W(s) = \frac{\left(k+\frac{1}{s}\right)\frac{10}{s(s+3)}}{1+\left(k+\frac{1}{s}\right)\frac{10}{s(s+3)}} = \frac{10(ks+1)}{s^2(s+3)+10(ks+1)} =$$

$$= \frac{10(ks+1)}{s^3+3s^2+10ks+10}$$

1.2)
$$\begin{array}{c|cc} 3 & 1 & 10k \\ 2 & 3 & 10 \\ 1 & \frac{30k-10}{3} & \\ 0 & 10 & \end{array}$$

ILUL stabile per $K > \frac{1}{3}$.

1.3)
$$Y_f(s) = W(s) = \frac{12s+10}{s^3+3s^2+12s+10} = \frac{12s+10}{(s+1)(s^2+2s+10)}$$

$$= \frac{A}{s+1} + \frac{B(s+1)+C\cdot 3}{(s+1)^2+(3)^2}$$

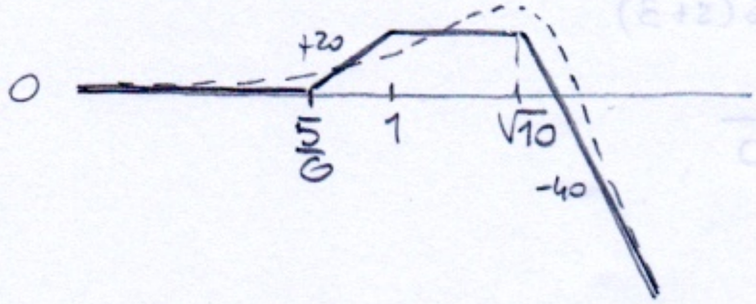
$$A(s^2+2s+10)+B(s+1)^2+C\cdot 3(s+1) = 12s+10$$

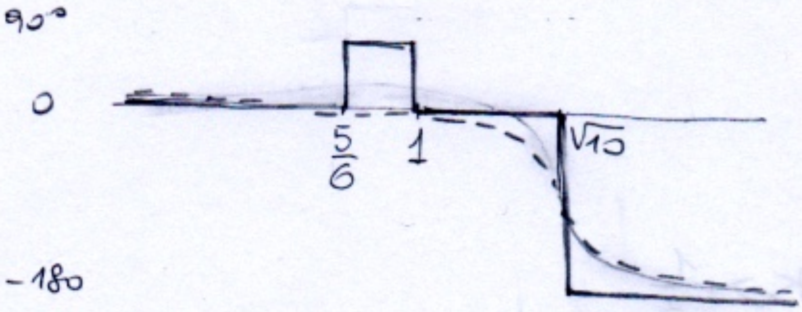
$$\begin{cases} A+B=0 \\ 2A+2B+3C=12 \\ 10A+B+3C=10 \end{cases} \qquad \begin{aligned} A&=-\tfrac{2}{9} \\ B&=\tfrac{2}{9} \\ C&=4 \end{aligned}$$

$$y_f(t) = \left\{-\frac{2}{9}e^{-t} + \frac{2}{9}e^{-t}\cos(3t) + 4e^{-t}\sin(3t)\right\}\cdot 1(t)$$

1.4)
$$W(s) = \frac{\cancel{10}\left(1+\frac{6}{5}s\right)}{(1+s)\cancel{10}\left(1+\frac{2}{10}s+\frac{1}{10}s^2\right)} = \frac{1+\frac{6}{5}s}{(1+s)\left(1+2\frac{1/\sqrt{10}}{\sqrt{10}}s+\frac{s^2}{10}\right)}$$

punti di rottura: $\frac{5}{6}$, 1, $\sqrt{10}$





2.1) $A = \begin{bmatrix} 0 & 1 \\ \frac{5}{36} & \frac{2}{3} \end{bmatrix}$

$\det(\lambda I - A) = \det \begin{bmatrix} \lambda & -1 \\ -\frac{5}{36} & \lambda - \frac{2}{3} \end{bmatrix} = \lambda^2 - \frac{2}{3}\lambda - \frac{5}{36} =$

$= \left(\lambda + \frac{1}{6}\right)\left(\lambda - \frac{5}{6}\right) \qquad \lambda_1 = -\frac{1}{6} \quad , \quad \lambda_2 = \frac{5}{6}$

modi: $\left(-\frac{1}{6}\right)^k \; , \; \left(\frac{5}{6}\right)^k$

Sistema asintoticamente stabile

2.2) $G(z) = [1 \quad 0] \begin{bmatrix} z & -1 \\ -\frac{5}{36} & z - \frac{2}{3} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\left(z + \frac{1}{6}\right)\left(z - \frac{5}{6}\right)}$

$Y_f(z) = \frac{1}{z}\left[\frac{z}{z-1} - \frac{z}{z - \frac{5}{6}}\right] = \frac{\frac{1}{6}}{(z-1)\left(z - \frac{5}{6}\right)}$

$U(z) = \frac{Y_f(z)}{G(z)} = \frac{\frac{1}{6}\left(z + \frac{1}{6}\right)}{z-1} = \frac{1}{6}\,\frac{z}{z-1} + \frac{1}{36}\,\frac{1}{z-1}$

$$u(k) = \frac{1}{6}\mathbb{1}(k) + \frac{1}{36}\mathbb{1}(k-1)$$

2.3) Poli: $e^{\pm j\frac{\pi}{2}} = \pm j$

$$\det(\lambda I - A - BF) = \lambda^2 + 1$$

$$A+BF = \begin{bmatrix} 0 & 1 \\ \frac{5}{36} & \frac{2}{3} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}[f_1 \quad f_2] = \begin{bmatrix} 0 & 1 \\ \frac{5}{36}+f_1 & \frac{2}{3}+f_2 \end{bmatrix}$$

$$\det\begin{pmatrix} \lambda & -1 \\ -\frac{5}{36}-f_1 & \lambda-\frac{2}{3}-f_2 \end{pmatrix} = \lambda^2 - \left(\frac{2}{3}+f_2\right)\lambda - \left(\frac{5}{36}+f_1\right)$$

$$\begin{cases} \frac{2}{3}+f_2 = 0 \\ \frac{5}{36}+f_1 = -1 \end{cases} \qquad F = \begin{bmatrix} -\frac{41}{36} & -\frac{2}{3} \end{bmatrix}$$

2.4) $$G(z) = \frac{1}{z^2 - \frac{2}{3}z - \frac{5}{36}}$$

i/o: $y(k+2) - \frac{2}{3}y(k+1) - \frac{5}{36}y(k) = u(k)$

3.1) $$G(s) = [1 \quad 0]\begin{bmatrix} s & -1 \\ 16 & s \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s^2+16}$$

L'ingresso $u(t) = \cos(4t)$ genera una $y_f(t)$ che contiene il modo divergente $t \cdot \cos(4t)$.

3.2) $$Y_\ell(s) = C(sI-A)^{-1}x(0) = [1 \quad 0]\begin{bmatrix} s & -1 \\ 16 & s \end{bmatrix}^{-1}\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} =$$

$$= \frac{s\,x_1(0) + x_2(0)}{s^2+16}$$

$$y_\ell(t) = x_1(0)\cdot\cos(4t) + \frac{x_2(0)}{4}\sin(4t)$$

Valori massimi $\Rightarrow \frac{d}{dt}y_\ell(t) = \emptyset$

$$\dot{y}_\ell(t) = -4x_1(0)\sin(4t) + x_2(0)\cos(4t)$$

Per $t = \frac{\pi}{16} + k\frac{\pi}{2}$ si ha

$$\dot{y}_\ell(t) = -4x_1(0)\sin\left(\frac{\pi}{4}\right) + x_2(0)\cos\left(\frac{\pi}{4}\right) =$$

$$= \frac{\sqrt{2}}{2}\left(-4x_1(0) + x_2(0)\right) = \emptyset$$

$\Rightarrow x(0)$ deve soddisfare $\boxed{x_2(0) = 4x_1(0)}$

3.3) $\begin{cases} \bar{x}_2 = 0 \\ -16\bar{x}_1 + 20\bar{x}_1 - \frac{1}{4}\bar{x}_1^3 - \bar{x}_2 = 0 \end{cases}$

$\begin{cases} \bar{x}_2 = 0 \\ 4\bar{x}_1 - \frac{1}{4}\bar{x}_1^3 = \emptyset \end{cases}$ $\quad \bar{x}_A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \bar{x}_B = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad \bar{x}_C = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

3.4) $J = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ 4 - \frac{3}{4}x_1^2 & -1 \end{bmatrix}$

$J_{|x=\bar{x}_A} = \begin{bmatrix} 0 & 1 \\ 4 & -1 \end{bmatrix}$ $\quad \det(\lambda I - J_A) = \lambda^2 + \lambda - 4$

$\bar{x}_A$ stato di equilibrio INSTABILE

$J_{|x=\bar{x}_B,\bar{x}_C} = \begin{bmatrix} 0 & 1 \\ -8 & -1 \end{bmatrix}$ $\quad \det(\lambda I - J_{|B,C}) = \lambda^2 + \lambda + 8$

$\bar{x}_B$, $\bar{x}_C$ stati di equilibrio ASINTOTICAMENTE STABILI.