Esercizio 1

1. $$A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix} \qquad B=\begin{bmatrix}0\\0\\1\end{bmatrix} \qquad C=\begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

$$\det(\lambda I-A)=\lambda^3-3\lambda-2=(\lambda+1)^2(\lambda-2)$$

Modi: $2^k\cdot \mathbb{1}(k)$; $(-1)^k\cdot\mathbb{1}(k)$; $k\cdot(-1)^k\cdot\mathbb{1}(k)$

$$Y_{imp}(z)=C(zI-A)^{-1}B=\frac{z+1}{z^3-3z-2}=\frac{1}{(z+1)(z-2)}=$$

$$=\frac{\frac{1}{3}}{z-2}-\frac{\frac{1}{3}}{z+1} \Rightarrow y_{imp}(k)=\left[\frac{1}{3}2^{k-1}-\frac{1}{3}(-1)^{k-1}\right]\mathbb{1}(k-1)$$

2. $$\mathcal{R}=\begin{bmatrix}B & AB & A^2B\end{bmatrix}=\begin{bmatrix}0 & 1 & 1+\alpha\\ 0 & 1 & 1+\alpha\\ 1 & \alpha & 2+\alpha^2\end{bmatrix}$$

$\det\mathcal{R}=0 \Rightarrow$ Il sistema non è completamente raggiungibile, $\forall\alpha$

$$X^r=L\left\{\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}1\\1\\\alpha\end{pmatrix}\right\}$$

3. $$\mathcal{O}=\begin{bmatrix}C\\ CA\\ CA^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 2 & 1 & 1+\alpha\end{bmatrix}$$

$\det\mathcal{O}=\alpha \Rightarrow$ Il sistema è completamente osservabile $\forall\alpha\neq 0$

Per $\alpha=0$ $$X^{no}=\ker\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 2 & 1 & 1\end{bmatrix}=L\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

4. $$\det(\lambda-A+LC)=\lambda^3$$

$$A-LC=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 1\end{bmatrix}-\begin{bmatrix}l_1\\ l_2\\ l_3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\end{bmatrix}=\begin{bmatrix}-l_1 & 1 & 1\\ 1-l_2 & 0 & 1\\ 1-l_3 & 1 & 1\end{bmatrix}$$

$$\det\begin{pmatrix} \lambda+l_1 & -1 & -1 \\ l_2-1 & \lambda & -1 \\ l_3-1 & -1 & \lambda-1 \end{pmatrix} = \lambda^3 + (l_1-1)\lambda^2 + (l_2+l_3-l_1-3)\lambda + \\ + l_3 - l_1 - 1$$

$$\begin{cases} l_1 - 1 = 0 \\ l_2+l_3-l_1-3=0 \\ l_3-l_1-1=0 \end{cases} \qquad \begin{aligned} l_1&=1 \\ l_2&=2 \\ l_3&=2 \end{aligned} \qquad L=\begin{pmatrix}1\\2\\2\end{pmatrix}$$

Esercizio 2

1. $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ Modi: $1\!\!1(t)$, $t\cdot 1\!\!1(t)$, $e^{-t}\cdot 1\!\!1(t)$
sistema instabile.

2. $X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}B\cdot U(s)$, $U(s)=1$

$X(s)=0 \implies x(0) = -B = \begin{pmatrix}0\\0\\1\end{pmatrix}$

3. $$\left.\begin{aligned} x_2&=0 \\ x_3&=0 \\ -x_3-1-x_2-x_1^3&=0 \end{aligned}\right\} \qquad \begin{aligned} x_2&=0 \\ x_3&=0 \\ x_1&=-1 \end{aligned} \qquad \bar{x}=\begin{pmatrix}-1\\0\\0\end{pmatrix}$$

4. $$\frac{\partial f}{\partial x} = \left.\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3x_1^2 & -1 & -1 \end{bmatrix}\right|_{x=\bar{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -1 & -1 \end{bmatrix} = J$$

$\det(\lambda I - J) = \lambda^3 + \lambda^2 + \lambda + 3$

$$\begin{array}{c|cc} 3 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & -2 & \\ 0 & 3 & \end{array}$$

$\bar{x}$ è stato di equilibrio <u>instabile</u>

Esercizio 3.

1. $$W(s) = \frac{G_1(s)\,P(s)}{1+\frac{1}{k}G_1(s)\,P(s)} \quad \text{dove} \quad P(s) = \frac{G_2(s)}{1+C(s)G_2(s)}$$

$$P(s) = \frac{\frac{1-s}{s+3}}{1+\frac{1-s}{s+3}\cdot\frac{1}{s+2}} = \frac{(1-s)(s+2)}{s^2+4s+7}$$

$$W(s) = \frac{\frac{10}{s}\cdot\frac{(1-s)(s+2)}{s^2+4s+7}}{1+\frac{1}{k}\,\frac{10}{s}\,\frac{(1-s)(s+2)}{s^2+4s+7}} = \frac{10(1-s)(s+2)}{s^3+4s^2+7s+\frac{10}{k}(1-s)(s+2)}$$

$$= \frac{10(1-s)(s+2)}{s^3+\left(4-\frac{10}{k}\right)s^2+\left(7-\frac{10}{k}\right)s+\frac{20}{k}}$$

2.

3	1	$7-\frac{10}{k}$
2	$4-\frac{10}{k}$	$\frac{20}{k}$
1	$\frac{\left(7-\frac{10}{k}\right)\left(4-\frac{10}{k}\right)-\frac{20}{k}}{4-\frac{10}{k}}$	
0	$\frac{20}{k}$	

$$\begin{cases} 4-\frac{10}{k}>0 \\ 28-\frac{130}{k}+\frac{100}{k^2}>0 \\ \frac{20}{k}>0 \end{cases}$$

$$k > \frac{10}{4}$$

$$28k^2-130k+100>0 \Rightarrow k > \frac{65+\sqrt{65^2-2800}}{28} \simeq 3{,}67$$

$\Rightarrow$ Il sistema è ILUL stabile per $k > \frac{65+\sqrt{1425}}{28}$

3. $$Y_f(s) = \frac{10(1-s)(s+2)}{s^3+2s^2+5s+4}\cdot\frac{1}{s} =$$

Ruffini! $\rightarrow$ $$= \frac{10(1-s)(s+2)}{(s+1)(s^2+s+4)\,s} =$$

$$= \frac{5}{s} - \frac{5}{s+1} - \frac{2\sqrt{15}\,\frac{\sqrt{15}}{2}}{s^2+s+4}$$

$\left\{ \begin{array}{l} s^2+s+4 = \\ = \left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{15}}{2}\right)^2 \end{array} \right.$

$$\Rightarrow \; y_f(t) = \left\{ 5 - 5e^{-t} - 2\sqrt{15}\, e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{15}}{2} t\right) \right\} \cdot \mathbb{1}(t)$$

4. $$W(s) = \frac{5(1-s)\left(1+\frac{1}{2}s\right)}{(1+s)\left(1+\frac{2\cdot\frac{1}{4}}{2}\cdot s + \frac{1}{2^2}s^2\right)}$$

punti di rottura	singolarità
1	zero reale positivo
1	polo reale negativo
2	zero reale negativo
2	poli complessi a parte reale negativa $\left(\zeta = \frac{1}{4}\right)$