1.I) $x_1(k) = y(k) \quad x_2(k) = y(k+1)$

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = \frac{4}{3} x_2(k) - \frac{1}{3} x_1(k) + u(k)$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{3} & \frac{4}{3} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

1.II) $x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$Y_\ell(z) = C(zI - A)^{-1} z\, x(0) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z & -1 \\ \frac{1}{3} & z - \frac{4}{3} \end{bmatrix}^{-1} z \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

$$= \frac{\left(z - \frac{4}{3}\right) z}{z^2 - \frac{4}{3} z + \frac{1}{3}} = \frac{3}{2} \cdot \frac{z}{z - \frac{1}{3}} - \frac{1}{2} \frac{z}{z-1}$$

$$y_\ell(k) = \left\{ \frac{3}{2} \left(\frac{1}{3} \right)^k - \frac{1}{2} \right\} \cdot \mathbb{1}(k)$$

1.III) $G(z) = C(zI - A)^{-1} B + D = \dfrac{1}{z^2 - \frac{4}{3} z + \frac{1}{3}}$

$$y_f(k) = 6 \quad \forall k \geq 2 \implies Y_f(z) = 6 \cdot \frac{z}{z-1} \cdot \frac{1}{z^2}$$

$$U(z) = \frac{Y_f(z)}{G(z)} = \frac{6}{\cancel{(z-1)} \cdot z} \cdot \cancel{(z-1)}\left(z - \frac{1}{3}\right) =$$

$$= 6 \left\{ 1 - \frac{1}{3} \cdot z^{-1} \right\}$$

$$u(k) = 6\,\delta(k) - 2\,\delta(k-1)$$

2.I) $Y_{imp}(s) = G(s)H(s) = \frac{10}{(s+5)^2(s+1)} =$

$$= \frac{5}{8} \cdot \frac{1}{s+1} - \frac{5}{2} \frac{1}{(s+5)^2} - \frac{5}{8} \frac{1}{s+5}$$

$$y_{imp}(t) = \left\{ \frac{5}{8} e^{-t} - \frac{5}{2} t e^{-5t} - \frac{5}{8} e^{-5t} \right\} \cdot \mathbb{1}(t)$$

2.II) $W(s) = \frac{G(s)}{1+G(s)C(s)} \cdot H(s) = \frac{10}{(s+5)^2(s+1)+k} =$

$$= \frac{10}{s^3 + 11s^2 + 35s + 25 + K}$$

$$\begin{array}{c|cc} 3 & 1 & 35 \\ 2 & 11 & 25+K \\ 1 & \frac{360-K}{11} & \\ 0 & 25+K & \end{array} \Rightarrow \left. \begin{array}{l} 360-K>0 \\ 25+K>0 \end{array} \right\} \Downarrow$$

$$K \in (-25, 360)$$

2.III) $4 \cdot |W(j)| < 1$

$$4 \cdot \left| \frac{10}{K+14+j\,34} \right| < 1$$

$$1600 < (K+14)^2 + 34^2$$

$$|K+14| > \sqrt{444}$$

$$\left\{ K < -14 - \sqrt{444} \lor K > -14 + \sqrt{444} \right\} \land K \in (-25, 360)$$

$$\Rightarrow \quad K \in (-14 + \sqrt{444}, 360)$$

3.I)

$$x_1(k+1) = 0{,}8\, x_1(k) + u(k)$$
$$x_2(k+1) = 0{,}1\, x_1(k) + 0{,}8\, x_2(k)$$
$$x_3(k+1) = 0{,}1\, x_2(k) + 0{,}9\, x_3(k)$$
$$y(k) = 0{,}1\,(x_1(k) + x_2(k) + x_3(k))$$

3.II)

$$G(z) = C(zI-A)^{-1}B + D =$$
$$= \begin{bmatrix} 0{,}1 & 0{,}1 & 0{,}1 \end{bmatrix} \begin{bmatrix} z-0.8 & 0 & 0 \\ -0{,}1 & z-0.8 & 0 \\ 0 & -0{,}1 & z-0{,}9 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} =$$
$$= \frac{0{,}1\,\{(z-0.8)(z-0.9) + 0.1\,(z-0{,}9) + 0{,}01\}}{(z-0.8)^2(z-0.9)} =$$
$$= \frac{0.1\, z^2 - 0{,}16\, z + 0{,}064}{(z-0{,}8)^2(z-0{,}9)} = 0.1\, \frac{\cancel{(z-0.8)^2}}{\cancel{(z-0.8)^2}(z-0{,}9)}$$

$$y(k+1) = 0.9\, y(k) + 0.1\, u(k)$$

3.III)

$$\lim_{k\to+\infty} x_f(k) = \lim_{z\to 1} (z-1)\,(zI-A)^{-1} B \cdot U(z)$$

$$U(z) = \frac{z}{z-1}$$

$$\lim_{z\to 1} (z-1)(zI-A)^{-1}B \cdot \frac{z}{z-1} = \lim_{z\to 1} \frac{\begin{bmatrix} (z-0{,}8)(z-0{,}9) \\ 0.1(z-0{,}9) \\ 0{,}01 \end{bmatrix}}{(z-0{,}8)^2(z-0{,}9)}$$

$$= \begin{bmatrix} 5 \\ 2.5 \\ 2.5 \end{bmatrix}$$

percentuale asintotica di dirigenti = 25 %

4.I) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -10 & -2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad C = [1 \;\; 0 \;\; 0]$

$D = 0$

$\det(\lambda I - A) = \lambda(\lambda^2 + 2\lambda + 10)$

poli: $\lambda_1 = 0 \qquad \lambda_{2,3} = -1 \pm j3$

modi: $1\!\!1(t)$ aperiodico, limitato non convergente

$\left.\begin{array}{l} e^{-t}\cos(3t) \\ e^{-t}\sin(3t) \end{array}\right\}$ pseudoperiodici, convergenti

4.II) $Y_\ell(s) = C(sI - A)^{-1}x(0) =$

$$= [1 \;\; 0 \;\; 0]\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 10 & s+2 \end{bmatrix}^{-1}\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} =$$

$$= \frac{(s^2 + 2s + 10)x_1(0) + (s+2)x_2(0) + x_3(0)}{s(s^2 + 2s + 10)}$$

Affinché $\lim_{t \to +\infty} y_\ell(t) = 0$ occorre ci sia uno zero in

$s = 0 \implies 10x_1(0) + 2x_2(0) + x_3(0) = \emptyset$

4.III) $Y_f(s) = [C(sI - A)^{-1}B + D] \cdot U(s) =$

$$= \frac{1}{s(s^2 + 2s + 10)}\left[\frac{1}{s+1} - \beta\frac{1}{s+2}\right]$$

$\lim_{t \to +\infty} y_f(t) = \lim_{s \to 0} sY_f(s) = \frac{1}{10}\left(1 - \frac{\beta}{2}\right)$

$-1 < \frac{1}{10}\left(1 - \frac{\beta}{2}\right) < 1$

$-10 < \frac{\beta}{2} - 1 < 10 \implies \beta \in (-18, 22)$