1.I) $W(s) = \dfrac{C(s)\,G(s)}{1 + K\,C(s)\,G(s)} = \dfrac{10}{s^2+6s+10K}$

modi convergenti : $K>0$ (Cartesio)
modi aperiodici : $\Delta = 36-40K > 0$
$\boxed{K \in \left(0, \frac{9}{10}\right)}$

1.II) $W(s) = \dfrac{\frac{10}{s(s+6)(s+2)}}{1 + K\frac{10}{s(s+6)(s+2)}} = \dfrac{10}{s^3+8s^2+12s+10K}$

Per l'esistenza del limite $W(s)$ deve avere poli a parte reale negative.

$$\begin{array}{c|cc} 3 & 1 & 12 \\ 2 & 8 & 10K \\ 1 & \frac{96-10K}{8} & \\ 0 & 10K & \end{array}$$

$\begin{cases} 96-10\cdot K > 0 \\ 10\cdot K > 0 \end{cases}$

$K \in \left(0, \frac{96}{10}\right)$

$\lim\limits_{t\to+\infty} y_f(t) = G(0) = \dfrac{1}{K} < \dfrac{1}{5} \Rightarrow K > 5$

$\boxed{K \in \left(5, \frac{96}{10}\right)}$

1.III) $W(s) = \dfrac{10}{s^3+(6+p)s^2+6ps+10}$

Affinche e^{-t} sia un modo del sistema, -1 deve essere radice del denominatore di $W(s)$:

$-1+(6+p)-6p+10 = 0 \Rightarrow \boxed{p=3}$

$s^3+9s^2+18s+10 = (s+1)(s^2+8s+10)$

$p_{2,3} = -4 \pm \sqrt{16-10} = -4 \pm \sqrt{6}$

<u>modi</u>: e^{-t}, $e^{(-4+\sqrt{6})t}$, $e^{(-4-\sqrt{6})t}$

2.I)

$$x_1(k+1) = (1-\alpha)x_1(k) + \beta x_2(k) + u(k)$$
$$x_2(k+1) = \alpha x_1(k) + (1-\beta-\gamma)x_2(k)$$
$$y(k) = \gamma x_2(k)$$

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{8} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & \frac{3}{8} \end{bmatrix} \quad D = 0$$

2.II)

$$\det(\lambda I - A) = \left(\lambda - \frac{1}{2}\right)^2 - \frac{1}{16} = \lambda^2 - \lambda + \frac{3}{16} = 0$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1 - \frac{3}{4}}}{2} = \begin{cases} \frac{3}{4} \\ \frac{1}{4} \end{cases}$$

$\underline{\text{modi}}$: $\left(\frac{3}{4}\right)^k \mathbb{1}(k)$, $\left(\frac{1}{4}\right)^k \mathbb{1}(k)$ entrambi $\underline{\text{convergenti}}$

2.III)

$$u(k) = N \cdot \delta(k)$$

$$G(z) = \begin{bmatrix} 0 & \frac{3}{8} \end{bmatrix} \begin{bmatrix} z - \frac{1}{2} & -\frac{1}{8} \\ -\frac{1}{2} & z - \frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{\frac{3}{16}}{\left(z - \frac{3}{4}\right)\left(z - \frac{1}{4}\right)}$$

$$Y(z) = G(z) \cdot N = \frac{3}{16} N \left[\frac{R_1}{z - \frac{3}{4}} + \frac{R_2}{z - \frac{1}{4}} \right]$$

$$\begin{cases} R_1 + R_2 = 0 \\ -\frac{1}{4}R_1 - \frac{3}{4}R_2 = 1 \end{cases} \Longrightarrow \begin{matrix} R_1 = 2 \\ R_2 = -2 \end{matrix}$$

$$y(k) = \frac{3}{8} N \left\{ \left(\frac{3}{4}\right)^{k-1} - \left(\frac{1}{4}\right)^{k-1} \right\} \cdot \mathbb{1}(k-1)$$

$$@\, k=3 \rightarrow \quad y(3) = \frac{3}{8} N \left\{ \frac{9}{16} - \frac{1}{16} \right\} = \frac{3}{16} N \geq 15$$

$$\Rightarrow \boxed{N \geq 80}$$

3.I) $G(z) = \frac{z}{z-1} - \frac{z}{z-\frac{1}{2}} = \frac{\frac{1}{2}z}{z^2 - \frac{3}{2}z + \frac{1}{2}}$

Rappresentazione i/o

$$y(k+2) - \frac{3}{2}y(k+1) + \frac{1}{2}y(k) = \frac{1}{2}u(k+1)$$

3.II) i/o : $y_\ell(k) - \frac{3}{2}y_\ell(k-1) + \frac{1}{2}y_\ell(k-2) = \emptyset$

$\mathcal{Z}$: $Y_\ell(z) - \frac{3}{2}\left[z^{-1}Y_\ell(z) + y(-1)\right] +$

$$+ \frac{1}{2}\left[z^{-2}Y_\ell(z) + y(-1)z^{-1} + y(-2)\right] = \emptyset$$

$$Y_\ell(z) = \frac{\frac{3}{2}y(-1) - \frac{1}{2}y(-1)z^{-1} - \frac{1}{2}y(-2)}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}} =$$

$$= \frac{\left[\frac{3}{2}y(-1) - \frac{1}{2}y(-2)\right]z^2 - \frac{1}{2}y(-1)z}{(z-1)\left(z-\frac{1}{2}\right)}$$

Affinché $\lim_{k\to+\infty} y_\ell(k) = 0$, occorre cancellare il polo in $z=1$:

$$\frac{3}{2}y(-1) - \frac{1}{2}y(-2) - \frac{1}{2}y(-1) = 0$$

$$\boxed{y(-1) - \frac{1}{2}y(-2) = \emptyset}$$

3.III) $W_f(z) = \frac{1}{z-\frac{1}{2}} = P(z)\cdot G(z)\cdot \frac{z}{z-1}$

$$P(z) = \frac{1}{z-\frac{1}{2}} \cdot \frac{z-1}{z} \cdot \frac{(z-1)\left(z-\frac{1}{2}\right)}{\frac{1}{2}z} = \frac{2(z-1)^2}{z^2}$$

$$p(k) = \mathcal{Z}^{-1}[P(z)] = \mathcal{Z}^{-1}\left[\frac{2z^2 - 4z + 2}{z^2}\right] =$$

$$= 2\delta(k) - 4\delta(k-1) + 2\delta(k-2)$$

4.I)
$$A=\begin{bmatrix}-3 & 0 & 6\\ 0 & -1 & 0\\ -1 & -1 & -1\end{bmatrix}$$

$$\det(\lambda I-A)=\det\begin{pmatrix}\lambda+3 & 0 & -6\\ 0 & \lambda+1 & 0\\ 1 & 1 & \lambda+1\end{pmatrix}=$$

$$=(\lambda+3)(\lambda+1)(\lambda+1)+6(\lambda+1)=$$

$$=(\lambda+1)(\lambda^2+4\lambda+9)$$

$\lambda_1=-1 \quad \lambda_{2,3}=-2\pm\sqrt{-5}=-2\pm j\sqrt{5}$

<u>modi</u>: $e^{-t}\,1\!\!1(t)$ aperiodico convergente

$\left.\begin{array}{l} e^{-2t}\cos(\sqrt{5}t)\,1\!\!1(t)\\ e^{-2t}\sin(\sqrt{5}t)\,1\!\!1(t)\end{array}\right\}$ pseudoperiodici convergenti

4.II) $G(s)=C(sI-A)^{-1}B+D=$

$$=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}s+3 & 0 & -6\\ 0 & s+1 & 0\\ 1 & 1 & s+1\end{bmatrix}^{-1}\begin{bmatrix}0\\5\\0\end{bmatrix}=\frac{-30}{(s+1)(s^2+4s+9)}$$

$$Y_{imp}(s)=\frac{\gamma_1}{s+1}+\frac{\gamma_2(s+2)}{(s+2)^2+5}+\frac{\gamma_3\cdot\sqrt{5}}{(s+2)^2+5}$$

$$\gamma_1(s^2+4s+9)+\gamma_2(s^2+3s+2)+\gamma_3\sqrt{5}(s+1)=-30$$

$$\begin{cases}\gamma_1+\gamma_2=0\\ 4\gamma_1+3\gamma_2+\sqrt{5}\,\gamma_3=0\\ 9\gamma_1+2\gamma_2+\sqrt{5}\,\gamma_3=-30\end{cases}\longrightarrow\begin{array}{l}\gamma_1=-5\\ \gamma_2=+5\\ \gamma_3=+\sqrt{5}\end{array}$$

$$y_{imp}(t)=\left\{-5e^{-t}+5e^{-2t}\cos(\sqrt{5}t)+\sqrt{5}\,e^{-2t}\sin(\sqrt{5}t)\right\}1\!\!1(t)$$

4.III) $y_{perm}(t) = M \cdot |G(j)| \cos(t + \measuredangle G(j))$

$$|G(j)| = \left| \frac{-30}{(j+1)(j^2+4j+9)} \right| = \left| \frac{-30}{(j+1)(8+4j)} \right| =$$

$$= \frac{30}{\sqrt{2} \cdot \sqrt{80}} = \frac{30}{4\sqrt{10}}$$

$$M|G(j)| > 1 \Rightarrow M \frac{15}{2\sqrt{10}} > 1 \Rightarrow \boxed{M > \frac{2\sqrt{10}}{15}}$$