Es. 1

I) $$W(z) = \frac{1}{z} \cdot \frac{\frac{z}{z-1}}{1+\frac{z}{z-1}} \cdot \frac{1}{z} = \frac{1}{z} \cdot \frac{1}{2z-1}$$

II) $$Y_f(z) = W(z) \cdot \frac{z}{z-1} = \frac{1}{2} \cdot \frac{1}{\left(z-\frac{1}{2}\right)(z-1)} =$$

$$= \frac{R_1}{z-1} + \frac{R_2}{z-\frac{1}{2}} \qquad \left.\begin{array}{l} R_1 + R_2 = 0 \\ -\frac{1}{2}R_1 - R_2 = \frac{1}{2} \end{array}\right\} \begin{array}{l} R_1 = 1 \\ R_2 = -1 \end{array}$$

$$y_f(k) = \left\{1 - \left(\frac{1}{2}\right)^{k-1}\right\} \cdot 1\!\!1(k-1)$$

III) $$Y_f(z) = \frac{1}{z(z-1)}$$

$$U(z) = \frac{Y_f(z)}{W(z)} = \frac{1}{z(z-1)} \cdot z(2z-1) =$$

$$= \frac{2z-1}{z-1} = 2 \cdot \frac{z}{z-1} - \frac{1}{z-1}$$

$$u(k) = 2 \cdot 1\!\!1(k) - 1\!\!1(k-1)$$

Es. 2

I) $G(s) = \dfrac{2}{s^2+(2-k)s+(k+1)}$

Modi convergenti (cartesio): $\left.\begin{array}{l} 2-k>0 \\ k+1>0 \end{array}\right\} k \in (-1,2)$

Modi pseudoperiodici: $\Delta = (2-k)^2 - 4(k+1) < 0$

$k^2 - 8k < 0 \Rightarrow k \in (0,8)$

Modi pseudop. convergenti per $\boxed{k \in (0,2)}$.

II) $\lim_{t\to+\infty} y_f(t)$ $\exists$ finito se $k \in (-1,2)$.

$\lim_{t\to+\infty} y_f(t) = G(0) = \dfrac{2}{k+1} > 1 \Rightarrow k < 1$

$\Rightarrow \boxed{k \in (-1,1)}$

III) La risposta di regime transitorio converge a zero se $k \in (-1,2)$.

$|G(j)| = \left| \dfrac{2}{-1+(2-k)j+k+1} \right| = \dfrac{2}{|k+j(2-k)|}$

$= \dfrac{2}{\sqrt{k^2+(2-k)^2}} > 1$

$k^2 + (2-k)^2 < 4$

$2k^2 - 4k < 0 \rightarrow \boxed{k \in (0,2)}$

Es. 3

I) $y(k) = u(k) + \frac{1}{2} y(k-1) + \frac{1}{32} y(k-2)$

$$G(z) = \frac{z^2}{z^2 - \frac{1}{2} z - \frac{1}{32}}$$

poli: $\dfrac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{1}{8}}}{2} = \dfrac{\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{3}{2}}}{2} = \frac{1}{4} \pm \frac{1}{4}\sqrt{\frac{3}{2}}$

modi: $\left(\frac{1}{4} + \frac{1}{4}\sqrt{\frac{3}{2}}\right)^k, \quad \left(\frac{1}{4} - \frac{1}{4}\sqrt{\frac{3}{2}}\right)^k$

aperiodici, convergenti

II) $x_1(k) = y(k-2)$

$x_2(k) = y(k-1)$

$x_1(k+1) = x_2(k)$

$x_2(k+1) = u(k) + \frac{1}{2} x_2(k) + \frac{1}{32} x_1(k)$

$y(k) = u(k) + \frac{1}{2} x_2(k) + \frac{1}{32} x_1(k)$

$$A = \begin{bmatrix} 0 & 1 \\ \frac{1}{32} & \frac{1}{2} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} \frac{1}{32} & \frac{1}{2} \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \end{bmatrix}$$

III) $\lim_{k \to +\infty} y_f(k) = \lim_{z \to 1} (z-1) \cdot G(z) \cdot P \cdot \frac{z}{z-1}$

$= G(1) \cdot P = \dfrac{1}{1 - \frac{1}{2} - \frac{1}{32}} \cdot P = \frac{32}{15} P = \frac{1}{2}$

$P = \frac{15}{64} \simeq 0.23 \text{ Kg}$

Es. 4

I)

$$A = \begin{bmatrix} 0 & 25 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$G(s) = C(sI-A)^{-1}B =$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s & -25 & 0 \\ 1 & s & -1 \\ 0 & 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} =$$

$$= \frac{100}{s(s^2+25)}$$

$$y_{imp}(t) = \mathcal{L}^{-1}\left[\frac{100}{s(s^2+25)}\right] \Rightarrow$$

$$\frac{100}{s(s^2+25)} = \frac{\gamma_0}{s} + \gamma_1 \frac{s}{s^2+25} + \frac{\gamma_2 5}{s^2+25}$$

$$\gamma_0(s^2+25) + \gamma_1 s^2 + \gamma_2 5 s = 100$$

$$\gamma_0 = 4, \quad \gamma_1 = -4, \quad \gamma_2 = 0$$

$$y_{imp}(t) = \{4 - 4\cos(5t)\}\mathbb{1}(t)$$

II)

$$C(sI-A)^{-1}x(0) = \frac{7}{s}$$

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s & -25 & 0 \\ 1 & s & -1 \\ 0 & 0 & s \end{bmatrix}^{-1} \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} =$$

$$= \frac{s^2 x_1(0) + 25 s\, x_2(0) + 25\, x_3(0)}{s(s^2+25)} = \frac{7}{s}$$

$$\Rightarrow x_1(0) = 7 \quad x_2(0) = 0 \quad x_3(0) = 7$$

III) Modi: $\mathbb{1}(t)$, $\cos(5t)$, $\sin(5t)$

$$U(s) = \frac{s}{(s+p)^2} \Rightarrow Y_f(s) = \frac{100}{(s+p)^2(s^2+25)} \quad p<0$$

$$\Rightarrow u(t) = \{e^{-pt} - p \cdot t\, e^{-pt}\} \cdot \mathbb{1}(t)$$