1.I) $x_1(k+1) = y(k+1) = x_2(k) + 6u(k)$

$x_2(k+1) = y(k+2) - 6u(k+1) \underset{\uparrow}{=} y(k) = x_1(k)$

dall'eq. i/o

$y(k) = x_1(k)$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = \emptyset$$

1.II) $Y_\ell(z) = z \cdot C(zI-A)^{-1} x(\emptyset) =$

$$= z \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z & -1 \\ -1 & z \end{bmatrix}^{-1} \begin{bmatrix} x_1(\emptyset) \\ x_2(\emptyset) \end{bmatrix} =$$

$$= z \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{\begin{bmatrix} z & 1 \\ 1 & z \end{bmatrix}}{z^2-1} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \frac{z^2 x_1(0) + z\, x_2(0)}{z^2-1}$$

Imporre $Y_\ell(z) = \gamma \cdot \frac{z}{z-1}$

$$\frac{\cancel{z^2} x_1(0) + \cancel{z} x_2(0)}{\cancel{(z-1)}(z+1)} = \gamma \cdot \frac{\cancel{z}}{\cancel{z-1}} \Rightarrow \underline{x_1(0) = x_2(0) = \gamma}$$

1.III) $u(k) = 1(k) \Longrightarrow U(z) = \frac{z}{z-1}$

$Y_f(z) = G(z) \cdot U(z)$

$G(z) = C\,(zI-A)^{-1} B + D =$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z & -1 \\ -1 & z \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \frac{6z}{z^2-1}$$

$$Y_f(z) = \frac{6z}{z^2-1} \cdot \frac{z}{z-1}$$

$$\frac{Y_f(z)}{z} = \frac{6z}{(z-1)^2(z+1)} = \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z+1}$$

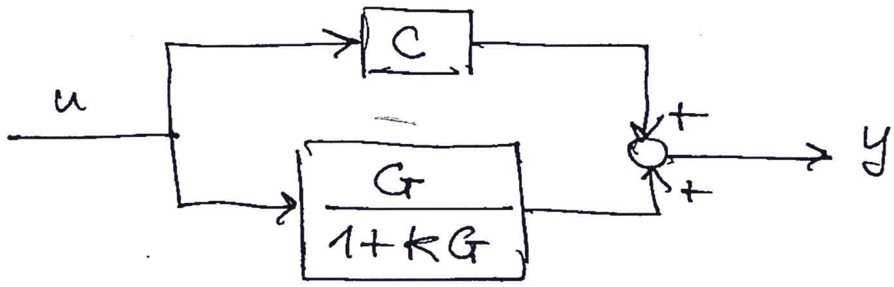
$$A(z^2-1)+B(z+1)+C(z-1)^2=6z$$

$$Az^2-A+Bz+B+Cz^2-2Cz+C=6z$$

$$\begin{cases} A+C=0 \\ B-2C=6 \\ -A+B+C=0 \end{cases} \qquad \begin{aligned} &A=-C \\ &B=2C+6 \\ &C+2C+6+C=\emptyset \end{aligned} \qquad \begin{aligned} &A=\tfrac{3}{2} \\ &B=3 \\ &C=-\tfrac{3}{2} \end{aligned}$$

$$y_f(k)=\frac{3}{2}\mathbb{1}(k)+3\cdot k\cdot\mathbb{1}(k)-\frac{3}{2}(-1)^k\cdot\mathbb{1}(k)$$

2.I)



$$W(s)=C(s)+\frac{G(s)}{1+kG(s)}=$$

$$=\frac{1}{s+1}+\frac{\frac{1}{s(s+4)}}{1+k\frac{1}{s(s+4)}}=$$

$$=\frac{1}{s+1}+\frac{1}{s(s+4)+k}=\frac{1}{s+1}+\frac{1}{s^2+4s+k}=$$

$$=\frac{s^2+4s+k+s+1}{(s+1)(s^2+4s+k)}=\frac{s^2+5s+k+1}{(s+1)(s^2+4s+k)}$$

2 II) Modi convergenti $\Rightarrow$ $k>0$
Modi aperiodici $\Rightarrow$ $4-k\geq 0$ $\Big\}\Rightarrow k\in(0,4]$

Per $k=3$: $(s+1)(s^2+4s+3)=(s+1)^2(s+3)$

modi: e^{-t}, te^{-t}, e^{-3t}

2. III) Limite errore finito $\Rightarrow$ $K > 0$

$$\lim_{t \to +\infty} y_f(t) = G(0) = \frac{K+1}{K} > 4$$

$$K+1 > 4K \Rightarrow 3K < 1 \Rightarrow K \in \left(0, \frac{1}{3}\right)$$

3.I)

$$x_1(k+1) = 0.6\, x_1(k) + 0.3\, x_3(k) + u(k)$$
$$x_2(k+1) = 0.9\, x_2(k) + 0.4\, x_1(k)$$
$$x_3(k+1) = 0.7\, x_3(k) + 0.1\, x_2(k)$$

3.II) $y(k) = x_1(k)$

$$G(z) = C(zI-A)^{-1}B + D =$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z-0.6 & 0 & -0.3 \\ -0.4 & z-0.9 & 0 \\ 0 & -0.1 & z-0.7 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} =$$

$$= \frac{(z-0.9)(z-0.7)}{(z-0.6)(z-0.9)(z-0.7) + 0.4(-0.1 \cdot 0.3)} =$$

$$= \frac{z^2 - 1.6z + 0.63}{(z-0.6)(z^2-1.6z+0.63) - 0.012} =$$

$$= \frac{z^2 - 1.6z + 0.63}{z^3 - 2.2z^2 + 1.59z - 0.39}$$

i/o:

$$y(k+3) - 2.2\,y(k+2) + 1.59\,y(k+1) - 0.39\,y(k) =$$
$$= u(k+2) - 1.6\,u(k+1) + 0.63\,u(k)$$

3.III) Il sistema ha un polo in $z = 1$!

$$\begin{array}{c|cccc|c} & 1 & -2.2 & 1.59 & & -0.39 \\ 1 & & 1 & -1.2 & & +0.39 \\ \hline & 1 & -1.2 & 0.39 & & \emptyset \end{array}$$

$$G(z) = \frac{z^2 - 1.6z + 0.63}{(z-1)\underbrace{(z^2 - 1.2z + 0.39)}}$$

le radici di questo polinomio sono interne al cerchio unitario

$$\lim_{k \to +\infty} y_f(k) = \lim_{z \to 1} (z-1) \cdot Y_f(z) =$$

$$= \lim_{z \to 1} (z-1) \cdot G(z) \cdot 100$$

perché l'ingresso è $u(k) = 100\,\delta(k) = \begin{cases} 100 & k = 0 \\ 0 & k > 0 \end{cases}$

$$= \lim_{z \to 1} \frac{z^2 - 1.6z + 0.63}{z^2 - 1.2z + 0.39} \cdot 100 =$$

$$= \frac{0.03}{0.19} \cdot 100 = \frac{3}{19} \cdot 100$$

4.I) $A = \begin{bmatrix} 0 & 1 \\ -5 & -1 \end{bmatrix}$

$$\det(\lambda I - A) = \det\begin{pmatrix} \lambda & -1 \\ 5 & \lambda+1 \end{pmatrix} = \lambda^2 + \lambda + 5$$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{-19}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{19}}{2}$$

modi: $e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{19}}{2}t\right)$, $e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{19}}{2}t\right)$

4.II) $G(s) = C(sI-A)^{-1}B + D =$

$$= \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} s & -1 \\ 5 & s+1 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} s+1 & 1 \\ -5 & s \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}{s^2+s+5} =$$

$$= \frac{s+2}{s^2+s+5}$$

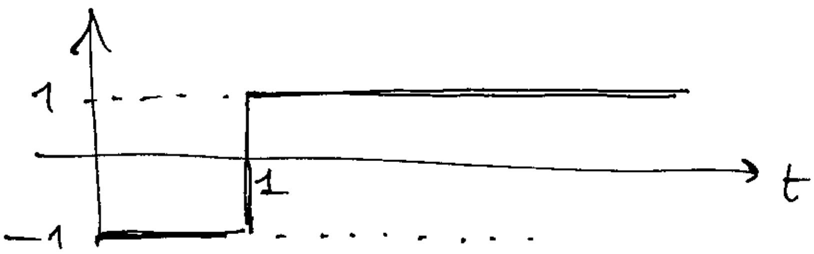
$u(t) = 10\cos(2t)$

$y_{perm}(t) = \underbrace{10 \cdot |G(j2)|}\cos(2t + \measuredangle G(j2))$

$$|G(j2)| = \left|\frac{j2+2}{-4+j2+5}\right| = \left|\frac{j2+2}{1+j2}\right| = \sqrt{\frac{8}{5}}$$

Ampiezza $= 10 \cdot \sqrt{\frac{8}{5}}$

4.III)



$u(t) = -1\!\!1(t) + 2 \cdot 1\!\!1(t-1)$

$U(s) = -\frac{1}{s} + 2 \cdot \frac{1}{s}e^{-s}$

$$Y_f(s) = G(s)U(s)$$

$$h(t) = \mathcal{L}^{-1}\left[G(s)\cdot\frac{1}{s}\right] = \mathcal{L}^{-1}\left[\frac{s+2}{s^2+s+5}\cdot\frac{1}{s}\right]$$

$$\Rightarrow \; y_f(t) = -h(t) + 2\, h(t-1)\cdot \mathbb{1}(t-1)$$

$$G(s)\frac{1}{s} = \frac{A}{s} + B\,\frac{s+\frac{1}{2}}{s^2+s+5} + C\,\frac{\frac{\sqrt{19}}{2}}{s^2+s+5}$$

$$A(s^2+s+5) + B\left(s+\frac{1}{2}\right)s + C\,\frac{\sqrt{19}}{2}\,s = s+2$$

$$A + B = 0 \qquad\qquad B = -\frac{2}{5}$$

$$A + \frac{1}{2}B + \frac{\sqrt{19}}{2}C = 1 \qquad\qquad C = \frac{2}{\sqrt{19}}\left(1-\frac{2}{5}+\frac{1}{5}\right) = \frac{8}{5\sqrt{19}}$$

$$5A = 2 \qquad\qquad A = \frac{2}{5}$$

$$h(t) = \frac{2}{5}\cdot\mathbb{1}(t) - \frac{2}{5}e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{19}}{2}t\right) + \frac{8}{5\sqrt{19}}e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{19}}{2}t\right)$$