Prova in itinere 20.11.2014

1.1) $G(z) = \dfrac{Y(z)}{U(z)}$

$$u(k) = 1\cdot\delta(k) + 2\,\delta(k-1) + 2\,\delta(k-2)$$

$$U(z) = 1\cdot z^0 + 2z^{-1} + 2z^{-2} = \frac{z^2+2z+2}{z^2}$$

$$y(k) = 1\cdot\delta(k) + 3\,\delta(k-1) + 5\cdot \mathbb{1}(k-2)$$

$$Y(z) = 1\cdot z^0 + 3\cdot z^{-1} + 5\cdot\frac{z}{z-1}\cdot\frac{1}{z^2} =$$

$$= \frac{z^2-z+3z-3+5}{z(z-1)} = \frac{z^2+2z+2}{z(z-1)}$$

$$G(z) = \frac{Y(z)}{U(z)} = \frac{\cancel{z^2+2z+2}}{\cancel{z}(z-1)}\cdot\frac{\cancel{z^2}}{\cancel{z^2+2z+2}} = \frac{z}{z-1}$$

1.2) $u(k) = \alpha\cdot\mathbb{1}(k) \rightarrow U(z) = \alpha\cdot\dfrac{z}{z-1}$

$$Y_f(z) = \alpha\cdot\frac{z^2}{(z-1)^2} = z\cdot\underbrace{\frac{\alpha z}{(z-1)^2}}_{\alpha k\cdot\mathbb{1}(k)}$$

$$y_f(k) = (k+1)\cdot\mathbb{1}(k)\cdot\alpha \leq 10 \quad \forall\, k\leq 300$$

$$301\cdot\alpha\leq 10 \qquad \boxed{\alpha\leq\frac{10}{301}}$$

1.3)

$$U(z) = u(0) + u(1)z^{-1}$$

$$Y(z) = y(0)$$

$$Y(z) = G(z)\cdot U(z) = \frac{z}{z-1}\cdot U(z)$$

$$\underline{\underline{y(0)}} = \frac{z}{z-1}\left(u(0)+u(1)z^{-1}\right) = \underbrace{\frac{u(0)z+u(1)}{z-1}}$$

$$u(1) = -u(0) \Rightarrow \frac{u(0)z-u(0)}{z-1} = u(0)$$

$$\underline{y(0) = u(0)}$$

2.1) $\boxed{y(k+2) - y(k) = u(k+2) + 4u(k+1)}$

$$x_1(k) = y(k) - u(k)$$
$$x_2(k) = \underline{y(k+1) - u(k+1)} - 4u(k)$$

$$x_1(k+1) = y(k+1) - u(k+1) = x_2(k) + 4u(k)$$
$$x_2(k+1) = y(k+2) - u(k+2) - 4u(k+1) = y(k) = x_1(k) + u(k)$$
$$y(k) = x_1(k) + \underline{u(k)}$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = 1$$

2.2) $$G(z) = \frac{z^2+4z}{z^2-1}$$

$$y(k) = \mathcal{Z}^{-1}\left[\frac{z^2+4z}{z^2-1}\right] \neq \cancel{\frac{R_1}{z+1} + \frac{R_2}{z-1}}$$

$$\frac{Y(z)}{z} = \frac{z+4}{z^2-1} = \frac{R_1}{z-1} + \frac{R_2}{z+1}$$

$$R_1 = \lim_{z \to 1} \frac{z+4}{z+1} = \frac{5}{2}$$

$$R_2 = \lim_{z \to -1} \frac{z+4}{z-1} = -\frac{3}{2}$$

$$Y(z) = \frac{5}{2}\frac{z}{z-1} - \frac{3}{2}\frac{z}{z+1}$$

$$y(k) = \frac{5}{2}\cdot \mathbb{1}(k) - \frac{3}{2}(-1)^k \cdot \mathbb{1}(k)$$

2.3) $u(k) = (\beta)^k \cdot \mathbb{1}(k) + (\beta)^{k-1}\, \mathbb{1}(k-1)$

$\lim_{k \to +\infty} y_f(k)$ [per quali β $\exists$ finito?]

$$U(z) = \frac{z}{z-\beta} + \frac{1}{z-\beta} = \frac{z+1}{z-\beta}$$

$$Y(z) = G(z)\cdot U(z) = \frac{z^2+4z}{z^2-1}\cdot\frac{z+1}{z-\beta} =$$

$$= \frac{z^2+4z}{(z-1)(z-\beta)}$$

$$|\beta| < 1 \longrightarrow \lim_{z \to 1} \cancel{(z-1)}\frac{z^2+4z}{\cancel{(z-1)}(z-\beta)} = \frac{5}{1-\beta}$$

$$\frac{5}{1-\beta} < 12 \qquad 5 < 12 - 12\beta$$

$$12\beta < 7 \qquad \beta < \frac{7}{12}$$

$$\longrightarrow \boxed{\beta \in \left(-1, \frac{7}{12}\right)}$$

3.1) $$G(s) = \frac{1}{s^2+2s+(5-k)}$$

Cartesio: $5-k>0 \rightarrow \boxed{k<5}$ modi convergenti

$$\frac{\Delta}{4} = \cancel{\quad} 1-(5-k) = k-4$$

$4 \le k < 5$ modi aperiodici convergenti

$k<4$ modi pseudoperiodici convergenti

3.2) $u(t) = \cos(t) \rightarrow y_{perm}(t) = M \cdot \sin(t)$

$$y_{perm}(t) = |G(j)| \cos(t + \angle G(j)) = M \underline{\sin(t)} =$$
$$= M \cos\left(t - \frac{\pi}{2}\right)$$

$$\angle G(j) = -\frac{\pi}{2}$$

$$G(j) = \frac{1}{j^2+2j+5-k} = \frac{1}{4-k+2j}$$

$\underline{k=4} \rightarrow G(j) = \frac{1}{2j} = -\frac{1}{2}j = \frac{1}{2}e^{-j\frac{\pi}{2}}$

$$M = \frac{1}{2}$$

Oppure:

$$\angle G(j) = -\angle 4-k+2j = -\operatorname{arctg}\frac{2}{4-k} = -\frac{\pi}{2}$$

$$\operatorname{arctg}\frac{2}{4-k} = \frac{\pi}{2} \qquad \frac{2}{4-k} = \tan\frac{\pi}{2} = +\infty$$

$\rightarrow k=4$

3.3) $u(t) = \mathbb{1}(t)$ $(k=1) \rightarrow G(s) = \dfrac{1}{s^2+2s+4}$

$$Y(s) = \frac{1}{s^2+2s+4} \cdot \frac{1}{s} \longrightarrow \mathbb{1}(t)$$

$$P_{1,2} = -1 \pm j\sqrt{3} \rightarrow \begin{matrix} e^{-t}\sin(\sqrt{3}t) \\ e^{-t}\cos(\sqrt{3}t) \end{matrix}$$

$$Y(s) = \frac{A}{s} + B\frac{\sqrt{3}}{(s+1)^2+3} + C\frac{s+1}{(s+1)^2+3}$$

$$A(s^2+2s+4) + B\sqrt{3}s + C(s+1)s = 1$$

$A + C = 0$ $\qquad C = -A = -\frac{1}{4}$

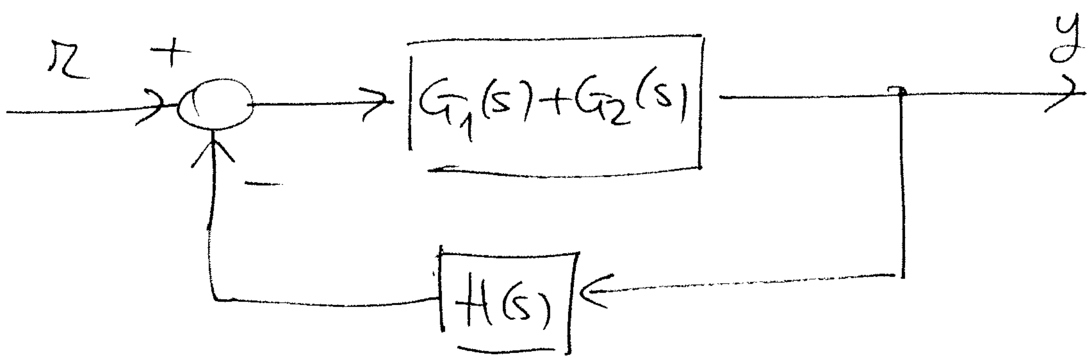
$2A + \sqrt{3}B + C = 0$

$4A = 1$ $\qquad A = \frac{1}{4}$

$$B = \frac{1}{\sqrt{3}}(-C-2A) = \frac{1}{\sqrt{3}}\left(\frac{1}{4} - \frac{1}{2}\right) = -\frac{1}{4} \cdot \frac{1}{\sqrt{3}}$$

$$y(t) = \frac{1}{4} \cdot \mathbb{1}(t) - \frac{1}{4\sqrt{3}} e^{-t}\sin(\sqrt{3}t) - \frac{1}{4} e^{-t}\cos(\sqrt{3}t)$$

4.1)



$$W(s) = \frac{G_1(s)+G_2(s)}{1+H(s)\cdot\left(G_1(s)+G_2(s)\right)} =$$

$$= \frac{\frac{1}{s+1}+\frac{1}{s+2}}{1+\frac{1}{2}\left(\frac{1}{s+1}+\frac{1}{s+2}\right)} = \frac{\frac{2s+3}{(s+1)(s+2)}}{1+\frac{1}{2}\,\frac{2s+3}{(s+1)(s+2)}} =$$

$$= \frac{2s+3}{s^2+3s+2+\frac{1}{2}(2s+3)} = \boxed{\frac{2s+3}{s^2+4s+\frac{7}{2}}}$$

4.2) <u>modi</u>

poli : $-2 \pm \sqrt{4-\frac{7}{2}} = -2 \pm \frac{1}{\sqrt{2}}$

modi: $e^{\left(-2+\frac{1}{\sqrt{2}}\right)t}$, $e^{\left(-2-\frac{1}{\sqrt{2}}\right)t}$

4.3) $\ddot{y}(t)+4\dot{y}(t)+\frac{7}{2}y(t) = 2\dot{r}(t)+3r(t)$

$$s^2Y_\ell(s)-sy(0)-\dot{y}(0)+4\left(sY_\ell(s)-y(0)\right)+\frac{7}{2}Y_\ell(s)=\emptyset$$

$$\left[s^2+4s+\frac{7}{2}\right]Y_\ell(s) = sy(0)+\mathring{y}(0)+4y(0)$$

$$Y_\ell(s) = \frac{sy(0)+\mathring{y}(0)+4y(0)}{s^2+4s+\frac{7}{2}}$$

$$sy(0)+\mathring{y}(0)+4y(0) = 2s+3$$

$y(0) = 2$

$\mathring{y}(0)+4y(0) = 3$ $\quad\quad$ $\mathring{y}(0) = 3-4y(0) = -5$