Problema 1

1) La funzione di trasferimento $G(s)$ da $r(t)$ a $y(t)$ è la trasformata di Laplace della risposta impulsiva:

$$G(s) = \mathcal{L}\left[-\frac{1}{3}e^{-2t} + \frac{2}{3}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{3}\right)\right] =$$

$$= \mathcal{L}\left[-\frac{1}{3}e^{-2t} + \frac{2}{3}e^{-\frac{t}{2}}\left[\cos\left(\frac{\pi}{3}\right)\cdot\cos\left(\frac{\sqrt{3}}{2}t\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\sqrt{3}}{2}t\right)\right]\right]$$

$$= -\frac{1}{3}\mathcal{L}\left[e^{-2t}\right] + \frac{1}{3}\mathcal{L}\left[e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\right] + \frac{1}{\sqrt{3}}\mathcal{L}\left[e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right]$$

$$= -\frac{1}{3}\cdot\frac{1}{s+2} + \frac{1}{3}\,\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{\sqrt{3}}\,\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} =$$

$$= \frac{-\frac{1}{3}}{s+2} + \frac{\frac{1}{3}\left(s+\frac{1}{2}\right)+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} =$$

$$= \frac{-\frac{1}{3}(s^2+s+1) + (s+2)\left(\frac{1}{3}s+\frac{2}{3}\right)}{(s+2)(s^2+s+1)} =$$

$$= \frac{\cancel{-\frac{1}{3}s^2} - \frac{1}{3}s - \frac{1}{3} + \cancel{\frac{1}{3}s^2} + \frac{2}{3}s + \frac{2}{3}s + \frac{4}{3}}{(s+2)(s^2+s+1)} =$$

$$= \frac{s+1}{(s+2)(s^2+s+1)}$$

Se $r(t) = 1(t) \longrightarrow R(s) = \frac{1}{s}$ e

$$Y_f(s) = G(s)\cdot R(s) = \frac{s+1}{(s+2)(s^2+s+1)}\cdot\frac{1}{s}$$

Poiché i modi sono: $\mathbb{1}(t)$, e^{-2t}, $e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)$, $e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)$

posso scrivere:

$$Y_f(s) = \frac{A}{s} + \frac{B}{s+2} + C\,\frac{s+\frac{1}{2}}{s^2+s+1} + D\,\frac{\frac{\sqrt{3}}{2}}{s^2+s+1} =$$

$$= \frac{A(s+2)(s^2+s+1) + B\,s(s^2+s+1) + C\left(s+\frac{1}{2}\right)(s^2+2s) + D\frac{\sqrt{3}}{2}(s^2+2s)}{s(s+2)(s^2+s+1)}$$

Imponendo l'uguaglianza dei numeratori

$$A\left[s^3+3s^2+3s+2\right] + B\left[s^3+s^2+s\right] + C\left[s^3+\frac{5}{2}s^2+s\right] + D\left[\frac{\sqrt{3}}{2}s^2+\sqrt{3}\,s\right] =$$
$$= s+1$$

$$\begin{cases} A+B+C = 0 \\ 3A+B+\frac{5}{2}C+\frac{\sqrt{3}}{2}D = 0 \\ 3A+B+C+\sqrt{3}D = 1 \\ 2A = 1 \end{cases} \qquad \begin{cases} B+C = -\frac{1}{2} \\ B+\frac{5}{2}C+\frac{\sqrt{3}}{2}D = -\frac{3}{2} \\ B+C+\sqrt{3}D = -\frac{1}{2} \\ A = \frac{1}{2} \end{cases} \quad \sqrt{3}D = 0$$

$$\begin{cases} B+C = -\frac{1}{2} \\ D = 0 \\ B+\frac{5}{2}C = -\frac{3}{2} \\ A = \frac{1}{2} \end{cases} \longrightarrow \frac{3}{2}C = -1 \qquad C = -\frac{2}{3}$$

$$B = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

$\Rightarrow$

$$y_f(t) = \left\{\frac{1}{2} + \frac{1}{6}e^{-2t} - \frac{2}{3}e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)\right\}\cdot \mathbb{1}(t)$$

2. La funzione di trasferimento da $r(t)$ a $y(t)$ dello schema a blocchi è pari a:

$$\frac{C(s)\cdot P(s)}{1 + C(s)\cdot P(s)\cdot H(s)} = G(s)$$

Posso ricavare P in funzione di G, C, H:

$$C\cdot P = (1 + CPH)G$$

$$CP = G + CPHG$$

$$CP - CPHG = G$$

$$(C - CHG)P = G$$

$$P = \frac{G}{C - CHG}$$

$$P(s) = \frac{\dfrac{s+1}{(s+2)(s^2+s+1)}}{\dfrac{1}{(s+1)^2} - \dfrac{1}{(s+1)^2}\cdot\dfrac{1}{\cancel{s+1}}\cdot\dfrac{\cancel{s+1}}{(s+2)(s^2+s+1)}} =$$

$$= \frac{\dfrac{s+1}{\cancel{(s+2)}\cancel{(s^2+s+1)}}}{\dfrac{(s+2)(s^2+s+1)-1}{(s+1)^2\cancel{(s+2)}\cancel{(s^2+s+1)}}} =$$

$$= \frac{(s+1)^3}{s^3+2s^2+s^2+2s+s+2-1} = \frac{(s+1)^3}{s^3+3s^2+3s+1} =$$

$$= \frac{(s+1)^3}{(s+1)^3} = 1$$

Problema 2

1. $$W(s) = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)\,H(s)} =$$

$$= \frac{K \cdot \dfrac{14s^2 - 15s + 1}{s^3 - 11s^2 + 18s}}{1 + K \cdot \dfrac{14s^2 - 15s + 1}{s^3 - 11s^2 + 18s} \cdot 1} =$$

$$= \frac{K \cdot (14s^2 - 15s + 1)}{s^3 - 11s^2 + 18s + K(14s^2 - 15s + 1)} =$$

$$= \frac{K(14s^2 - 15s + 1)}{s^3 + (14K - 11)s^2 + (18 - 15K)s + K}$$

$$\mathcal{L}[w(t)] = \mathcal{L}\left[(At^2 + Bt + C)e^{\alpha t}\right] =$$

$$= A \cdot \frac{2}{(s-\alpha)^3} + B\,\frac{1}{(s-\alpha)^2} + C\,\frac{1}{s-\alpha} = \frac{--}{(s-\alpha)^3}$$

Imponendo che $\mathcal{L}[w(t)] = W(s)$ si ha, uguagliando i denominatori

$$s^3 + (14K - 11)s^2 + (18 - 15K)s + K = (s - \alpha)^3 =$$

$$= s^3 - 3\alpha s^2 + 3\alpha^2 s - \alpha^3$$

$$\begin{cases} 14K - 11 = -3\alpha \\ 18 - 15K = 3\alpha^2 \\ K = -\alpha^3 \end{cases} \longrightarrow \begin{array}{l} -14\alpha^3 - 11 = -3\alpha \\ 14\alpha^3 - 3\alpha + 11 = 0 \end{array}$$

$\alpha = -1 \quad K = 1 \Rightarrow$ Nella seconda equazione:

$$18 - 15 \cdot 1 = 3(-1)^2 \quad ✓$$

Sostituendo nei rispettivi numeratori:

$$2A + B(s-\alpha) + C(s-\alpha)^2 = k(14s^2 - 15s + 1)$$

$$2A + B(s+1) + C(s+1)^2 = 14s^2 - 15s + 1$$

$$\underline{2A} + \underline{Bs} + \underline{B} + \underline{Cs^2} + \underline{2Cs} + \underline{C} = \underline{14s^2} - \underline{15s} + 1$$

$$\begin{cases} C = 14 \\ B + 2C = -15 \\ 2A + B + C = 1 \end{cases}$$

$$B = -43$$

$$A = \frac{1}{2}(1 - B - C) = 15$$

Problema 5

$G_1(s)$ ha due poli a parte reale negativa (regola di Cartesio) $\rightarrow$ ok

$G_2(s)$ ha un polo doppio in -1 $\rightarrow$ ok

$G_3(s)$ ha denominatore:

$$s^3+s^2+s+1 = (s+1)(s^2+1)$$

quindi i poli sono: $-1, \pm i$

$\Rightarrow$ il sistema ha modi e^{-t}, $\cos(t)$, $\sin(t)$ per cui la $y_T(t) \not\to \emptyset$

1) $u_1(t) = \sin(t)$, $G_1(s)$

$$y_p(t) = M \cdot |G_1(j\omega)| \cdot \sin(t + \measuredangle G_1(j\omega))$$ con $M=1$, $\omega = 1$

$$G_1(s) = \frac{s+1}{s^2+s+1}$$

$$G_1(j \cdot 1) = \frac{j+1}{\cancel{j^2}+j+\cancel{1}} = \frac{j+1}{j} = 1-j$$

$$|G_1(j)| = |1-j| = \sqrt{1+1} = \sqrt{2}$$

$$\measuredangle G_1(j) = \arctan \frac{-1}{1} = -\frac{\pi}{4}$$

$$y_p(t) = \sqrt{2} \sin\left(t - \frac{\pi}{4}\right)$$

...

per $u_3(t) = 1 + \cos\left(\frac{1}{2}t + \frac{\pi}{3}\right) = \cos(\emptyset \cdot t) + \cos\left(\frac{1}{2}t + \frac{\pi}{3}\right)$

$$y_p(t) = G(\emptyset) \cdot 1(t) + \left|G\left(j \cdot \tfrac{1}{2}\right)\right| \cos\left(\tfrac{1}{2}t + \tfrac{\pi}{3} + \measuredangle G\left(j \tfrac{1}{2}\right)\right)$$