Es. 1

I) $$W_1(s) = \frac{C(s)\cdot \frac{G(s)}{1+HG(s)}}{1+K\,C(s)\,\frac{G(s)}{1+HG(s)}} = \frac{C(s)G(s)}{1+HG(s)+K\,C(s)G(s)}$$

$$= \frac{10}{s^3+2s^2+Hs+K\cdot 10}$$

$$W_2(s) = \frac{1}{C(s)}\cdot W_1(s) = \frac{s}{s^3+2s^2+Hs+K\cdot 10}$$

II)

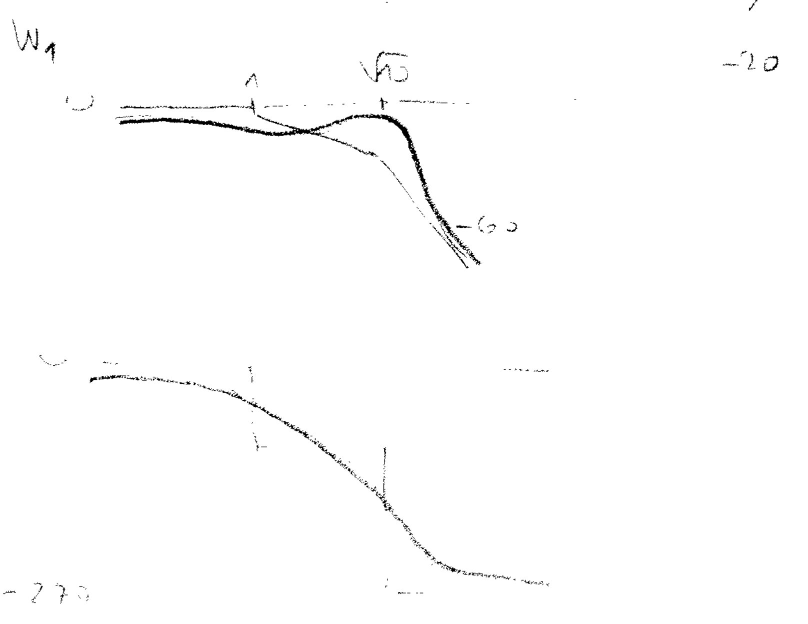
$$\begin{array}{ll} 1 & H \\ 2 & 10K \\ 2H-10K & 0 \\ 10K & \end{array}$$

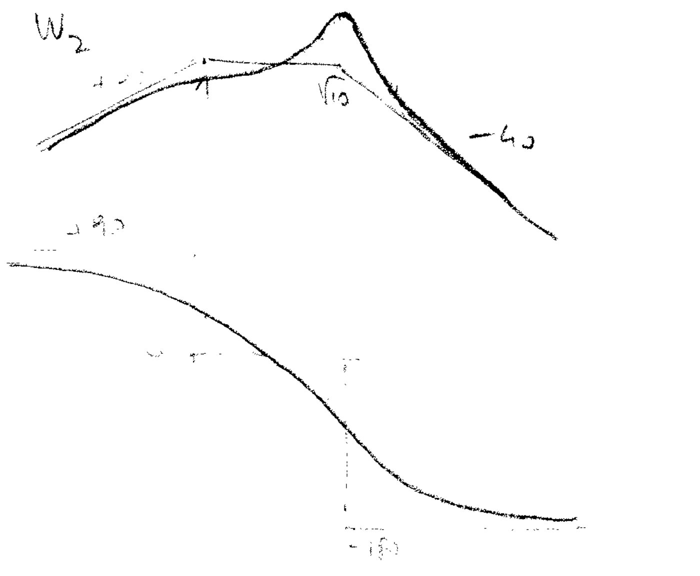
$$\begin{cases} 2H-10K>0 \\ K>0 \end{cases} \rightarrow \boxed{\begin{array}{l} K>0 \\ H>5K \end{array}}$$

III) $$W_1(s) = \frac{10}{s^3+2s^2+11s+10} = \frac{10}{(s+1)(s^2+s+10)} =$$

$$= \frac{1}{(1+s)\left(1+\frac{2}{\sqrt{10}}\left(\frac{1}{2\sqrt{10}}\right)s+\frac{s^2}{(\sqrt{10})^2}\right)}$$

$$W_2(s) = \frac{s}{(s+1)(s^2+s+10)} = \frac{s}{10}\cdot\frac{1}{(1+s)\left(1+\frac{2}{\sqrt{10}}\left(\frac{1}{2\sqrt{10}}\right)s+\frac{s^2}{(\sqrt{10})^2}\right)}$$





iv) $$W_2(j2) = \frac{j2}{-8j - 8 + 22j + 10} = \frac{2j}{14j + 2} =$$

$$= \frac{j}{1 + 7j} = \frac{7 + j}{50} = \frac{1}{\sqrt{50}} e^{j\,\mathrm{atan}\left(\frac{1}{7}\right)}$$

$$y_{perm}(t) = \frac{1}{\sqrt{50}} \cos\left(2t + \mathrm{atan}\left(\frac{1}{7}\right)\right)$$

Es. 2

$$A = \begin{pmatrix} 0.8 & i_d & 0 \\ 0.2 & +(1-i_d-i_r) & 0.2 \\ 0 & i_r & 0.8 \end{pmatrix}$$

I) $i_d = i_r = 0.3$

$\mathrm{eig}(A) = \{1,\ 0.8,\ 0.2\} \rightarrow$ marg. stab.

$$\lim_{K\to+\infty} x_\ell(k) = \lim_{z\to 1} (z-1)\, z \cdot (zI - A)^{-1} x(0) =$$

$$= \lim_{z\to 1} (z-1)z \begin{pmatrix} z-0.8 & -0.3 & 0 \\ -0.2 & z-0.4 & -0.2 \\ 0 & -0.3 & z-0.8 \end{pmatrix}^{-1} \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} =$$

$$= \lim_{z\to 1} \frac{z}{(z-0.8)(z-0.2)} \begin{pmatrix} z^2-1.2z+0.26 & 0.3(z-0.8) & 0.06 \\ 0.2(z-0.8) & (z-0.8)^2 & 0.2(z-0.8) \\ 0.06 & 0.3(z-0.8) & z^2-1.2z+0.26 \end{pmatrix} \begin{pmatrix} \vdots \end{pmatrix}$$

$$= \frac{1}{0.16} \begin{pmatrix} 0.06 & 0.06 & 0.06 \\ 0.04 & 0.04 & 0.04 \\ 0.06 & 0.06 & 0.06 \end{pmatrix} \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} = \begin{pmatrix} \frac{0.06}{0.16} \\ \frac{0.04}{0.16} \\ \frac{0.06}{0.16} \end{pmatrix} = \begin{pmatrix} 37.5\% \\ 25\% \\ 37.5\% \end{pmatrix}$$

e vale $\forall\, x(0)\ :\ \sum x_i(0) = 1$

II) $C = (1 \quad 0 \quad 0)$

$$A = \begin{pmatrix} 0.8 & i_d & 0 \\ 0.2 & 0.5 & 0.2 \\ 0 & 0.5-i_d & 0.8 \end{pmatrix}$$

$$\lim_{z\to 1} (z-1)z\, C\, (zI - A)^{-1} = \lim_{z\to 1} \frac{z(z-1)C}{\det} \begin{pmatrix} z-0.8 & -i_d & 0 \\ -0.2 & z-0.5 & -0.2 \\ 0 & i_d-0.5 & z-0.8 \end{pmatrix}'$$

$$= \lim_{z\to 1} \frac{z(z-1)}{\det} \begin{bmatrix} z^2-1{,}3z+0.4+0.2i_d-0.1 & i_d(z-0.8) & 0.2i_d \end{bmatrix}$$

$$= \frac{z(z-1)}{\det} \begin{bmatrix} 0{,}2\,i_d & 0.2\,i_d & 0{,}2\,i_d \end{bmatrix}$$

$$\det = [z-0.8]\left(z^2-1.3z+0.3+0{,}2\,i_d\right) - 0{,}2\,i_d\,(z-0.8) =$$
$$= (z-0.8)\left(z^2-1{,}3z+0.3\right) = (z-0.8)(z-1)(z-0.3)$$

$$\lim_{z\to 1} \ldots = \frac{1}{0{,}14}\cdot 0.2\,i_d > 0.5 \qquad \underline{i_d > 0{,}35}$$

III) $A^4 x(0) = \begin{pmatrix} 0.4892 \\ \vdots \\ \end{pmatrix}$ $\qquad A^5 x(0) = \begin{pmatrix} 0.5058 \\ * \\ * \end{pmatrix}$ $\qquad k = 5$

IV) $C = [1 \quad 0 \quad 0]$

$$\Theta = \begin{bmatrix} 1 & 0 & 0 \\ 0.8 & i_d & 0 \\ * & * & 0{,}2\,i_d \end{bmatrix} \longrightarrow \quad i_d \neq 0$$

se $i_d = 0 \qquad X^{\bar{o}} = L\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}$

$C = [0 \quad 1 \quad 0]$

$$\Theta = \begin{bmatrix} 0 & 1 & 0 \\ 0.2 & 1-i_d-i_r & 0{,}2 \\ 0{,}16+0{,}2(1-i_d-i_r) & * & 0{,}16+0{,}2(1-i_d-i_r) \end{bmatrix}$$

non oss. $\forall i_d, i_r$!

$$X^{\bar{o}} = L\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Es. 3

I)

$$\begin{cases} \dfrac{x_1 x_2}{x_2+4} - \dfrac{1}{2} = 0 \\ -\beta \dfrac{x_1 x_2}{x_2+4} + \dfrac{1}{2}(1-x_2) = 0 \end{cases} \qquad \begin{cases} x_1 x_2 = \dfrac{1}{2} x_2 + 2 \\ -\dfrac{\beta}{2} + \dfrac{1}{2} - \dfrac{1}{2} x_2 = 0 \end{cases}$$

$$x_2 = 1-\beta$$

$$x_1 = \frac{\frac{1}{2}x_2 + 2}{x_2} = \frac{\frac{1}{2}(1-\beta)+2}{1-\beta} = \frac{5-\beta}{2(1-\beta)} \qquad \beta \neq 1,5$$

II)

$$\frac{\partial f}{\partial x} = \left[\begin{matrix} \dfrac{x_2}{x_2+4} & \dfrac{x_1(x_2+4) - x_1 x_2}{(x_2+4)^2} \\ -\beta \dfrac{x_2}{x_2+4} & -\beta \dfrac{4x_1}{(x_2+4)^2} - \dfrac{1}{2} \end{matrix}\right]\Bigg|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2}} =$$

$$= \left[\begin{matrix} \dfrac{1-\beta}{5-\beta} & 2 \cdot \dfrac{5-\beta}{1-\beta} \cdot \dfrac{1}{(5-\beta)^2} \\ -\beta \dfrac{1-\beta}{5-\beta} & -\beta \cdot 2 \dfrac{1}{(1-\beta)(5-\beta)} - \dfrac{1}{2} \end{matrix}\right]$$

$$\begin{cases} \dfrac{1-\beta}{5-\beta} - 2\beta \dfrac{1}{(1-\beta)(5-\beta)} - \dfrac{1}{2} < 0 \\ -\dfrac{2\beta}{(5-\beta)^2} - \dfrac{1}{2}\dfrac{1-\beta}{5-\beta} + 2\beta \dfrac{1}{(5-\beta)^2} > 0 \end{cases}$$

$$\begin{cases} \dfrac{(1-\beta)^2 - 2\beta - \frac{1}{2}(5-6\beta+\beta^2)}{(1-\beta)(5-\beta)} < 0 \\ \dfrac{1-\beta}{5-\beta} < 0 \end{cases}$$

$$\begin{cases} \dfrac{1+\beta^2-2\beta-2\beta-\frac{5}{2}+3\beta-\frac{1}{2}\beta^2}{(1-\beta)(5-\beta)} < 0 \\ \dfrac{1-\beta}{5-\beta} < 0 \end{cases}$$

$$\begin{cases} \frac{1}{2}\beta^2-\beta-\frac{3}{2} > 0 \\ \dfrac{1-\beta}{5-\beta} < 0 \end{cases} \Rightarrow \begin{cases} \frac{1}{2}(\beta^2-2\beta-3) = \frac{1}{2}(\beta-3)(\beta+1) > 0 \\ \dfrac{1-\beta}{5-\beta} < 0 \end{cases}$$

$$\begin{cases} \beta > 3 \ \wedge \ \beta < -1 \\ 1 < \beta < 5 \end{cases} \Rightarrow \beta \in (3,5) \text{ as. stab.}$$

$$\beta = 3 \rightarrow A = \begin{bmatrix} -1 & -\frac{1}{2} \\ 3 & 1 \end{bmatrix} \qquad (\lambda+1)(\lambda-1)+\frac{3}{2} = \lambda^2+\frac{1}{2}$$

non si può decidere!

$\beta < 3 \wedge \beta > 5$ inst.

$\beta \neq 1$

III)

$$A = \begin{bmatrix} -3 & -\frac{2}{3} \\ 12 & \frac{13}{6} \end{bmatrix}$$

$$(\lambda+3)\left(\lambda-\frac{13}{6}\right)+8 = \lambda^2+\frac{5}{6}\lambda+\frac{3}{2} \qquad \lambda = \frac{-\frac{5}{6} \pm \sqrt{\frac{25}{36}-6}}{2} =$$

$$= -\frac{5}{12} \pm j\,\frac{\sqrt{191}}{12}$$

modi:

$$e^{-\frac{5}{12}t}\cos\left(\frac{\sqrt{191}}{12}t\right)$$

$$e^{-\frac{5}{12}t}\sin\left(\frac{\sqrt{191}}{12}t\right)$$

Es. 4

$$A = \begin{bmatrix} 0 & 1 & \alpha \\ \alpha & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

I) $$\mathcal{R} = \begin{bmatrix} 0 & \alpha & 1-\alpha \\ 0 & 1 & \alpha^2-1 \\ 1 & -1 & 1 \end{bmatrix} \qquad \det \mathcal{R} = \alpha^3 - \alpha + \alpha - 1 = 0$$

$$\alpha = 1$$

$$\mathcal{R}_{|\alpha=1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \qquad \mathcal{X}^r = L\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

II) $$T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \qquad \bar{A} = T^{-1}AT = \left[\begin{array}{cc|c} 0 & 1 & 1 \\ 1 & 0 & 1 \\ \hline 0 & 0 & -1 \end{array}\right] \qquad \bar{B} = T^{-1}B = \left[\begin{array}{c} 0 \\ 1 \\ \hline 0 \end{array}\right]$$

$$\lambda_r = \{1, -1\} \qquad \lambda_F = -1$$

III) $$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$F = -\mathrm{acker}(A, B, [0\ 0\ 0]) = [0 \quad 0 \quad 1]$$

$$A + BF = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x(k) = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \dots \right\}$$

IV) $$\mathcal{R}^{-1} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} u(2) \\ u(1) \\ u(0) \end{pmatrix}$$

V) $$\mathcal{R}_4^T (\mathcal{R}_4 \mathcal{R}_4^T)^{-1} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1/2 \\ -1/2 \end{pmatrix} = \begin{pmatrix} u(3) \\ u(2) \\ u(1) \\ u(0) \end{pmatrix}$$