(B) SLAM with simulated range measurements with respect to point landmarks and unknown data association

A key problem in robot navigation based on landmarks is to uniquely identify a perceived landmark. Artificial landmarks can be designed in order to make this identification easy (like QR codes, AprilTags, etc.). However, in natural environment commonly employed landmarks, like corners or furnitures, are often indistinguishable. Therefore, it is important to: (i) establish if a detected landmark is a new element to be added to the map or it is among those already present in the map; (ii) in the latter case, correctly associate the detected landmark to the existing one. Making errors in this case can dramatically deteriorate the performance of the SLAM algorithm. If associations are not detected, the map will be populated of many spurious landmarks and the robot will eventually be unable to correct the odometry errors using the environmental measurements. On the other hand, wrong associations will generate incorrect map estimates, with respect to which it will be impossible to localize the robot. Hence, data association is a crucial task that must be performed carefully.

Data association with Mahalanobis distance

A common method for performing data association is based on maximum likelihood. The idea is to compute, for each pair of range-angle measurements at time t, the likelihood that such measurements are associated to one of the previously detected landmarks. If the resulting maximum likelihood is above a certain threshold, it will be associated to the corresponding landmark. Conversely, if all likelihoods are below another threshold, the measurement is used to insert a new landmark in the map. Notice that when the maximum likelihood lies between the two thresholds, the measurement is simply discarded. In fact, it is better to avoid processing a measurement when its data association has a significant probability to be incorrect.

Consider a vector $x \in \mathbb{R}^n$ and a Gaussian distribution in $\mathbb{R}^n$ with mean μ and covariance matrix P. Then, by observing that $P^{-1/2}(x - \mu)$ is a standard Gaussian vector (with zero mean and unitary covariance), one has that $d = (x - \mu)' P^{-1}(x - \mu)$ is a χ^2 random variable with n degrees of freedom. The variable d is also called the *Mahalanobis distance* of vector x from the distribution $\mathcal{N}(\mu, P)$.

By using the above idea, we can compute the Mahalanobis distance of the measurements $m_{\rho_j}(t), m_{\alpha_j}(t)$ from its predicted distribution at time t provided by the EKF, assuming the measurement is associated to landmark L_k. This amounts to

$$d_{j,k}(t) = \delta_{j,k}(t)'[H_k(t)P(t|t-1)H_k'(t) + R]^{-1}\delta_{j,k}(t)$$

where

$$\delta_{j,k}(t) = \begin{bmatrix} m_{\rho_j}(t) - \sqrt{(\hat{L}_{k,x}(t|t-1) - \hat{x}(t|t-1))^2 + (\hat{L}_{k,y}(t|t-1) - \hat{y}(t|t-1))^2} \\ m_{\alpha_j}(t) - \text{atan2}\{\hat{L}_{k,y}(t|t-1) - \hat{y}(t|t-1), \hat{L}_{k,x}(t|t-1) - \hat{x}(t|t-1)\} + \hat{\theta}(t|t-1) \end{bmatrix},$$

$P(t|t-1)$ is the predicted covariance matrix of the EKF, and $H_k(t)$ is the Jacobian matrix of the nonlinear measurement model (1)-(2), computed in the predicted state values $\hat{x}(t|t-1)$, $\hat{y}(t|t-1)$, $\hat{\theta}(t|t-1)$, $\hat{L}_{k,x}(t|t-1)$, $\hat{L}_{k,y}(t|t-1)$.

Now, let

$$d_j^* = \min_k d_{j,k}$$

where the minimum is taken with respect to all landmark indexes k that are present in the map at time $t-1$. Then, by choosing two suitable thresholds $\tau_1 < \tau_2$, corresponding to the probability levels for association to an existing landmark and generation of a new landmark, respectively, one has that

- if $d_j^* < \tau_1$, the measurement $m_{\rho_j}(t), m_{\alpha_j}(t)$ is associated to landmark $L_{k_j^*}$, where

$$k_j^* = \arg\min_k d_{j,k}$$

- if $d_j^* > \tau_2$, the measurement $m_{\rho_j}(t), m_{\alpha_j}(t)$ is associated to a new landmark to be inserted in the map. The predicted state vector is augmented by inserting the new landmark position

$$\hat{L}_x = \hat{x}(t|t-1) + m_{\rho_j}(t)\cos\left(m_{\alpha_j}(t) + \hat{\theta}(t|t-1)\right)$$
$$\hat{L}_y = \hat{y}(t|t-1) + m_{\rho_j}(t)\sin\left(m_{\alpha_j}(t) + \hat{\theta}(t|t-1)\right)$$

while the covariance matrix $P(t|t-1)$ is augmented by inserting the covariance matrix of the new landmark (usually chosen as $\lambda I_{2\times 2}$, with λ a sufficiently large value).

Finally, in both cases the measurement $m_{\rho_j}(t), m_{\alpha_j}(t)$ is processed in the correction step of the EKF, with the established landmark association. The procedure is repeated for every measurement pair $m_{\rho_j}(t), m_{\alpha_j}(t)$, $j = 1, \ldots, \ell_t$, where ℓ_t is the number of landmarks detected by the Lidar at time t.

As an example, the values of the thresholds can be chosen as $\tau_1 = 5.9915$ and $\tau_2 = 13.8155$, corresponding respectively to the 95% and the 99.9% confidence levels of the χ^2 distribution with 2 degrees of freedom. This means that when a new landmark is inserted in the map, there is a probability of about 0.1% that the considered measurement refers to an existing landmark. Different thresholds can be chosen by using the inverse χ^2 distribution (in Python, use function `chi2.ppf` from `scipy.stats.distributions`).

Objective

Design and implement an Extended Kalman Filter providing an estimate of the robot pose and landmark positions, at every time t, based on the range measurements $m_{\rho_j}(t), m_{\alpha_j}(t)$ and odometric data $\hat{u}_f(t), \hat{u}_a(t)$. Insert the landmarks progressively into the state vector, when they are detected for the first time. For comparison with the ground truth, assume the initial condition of the EKF equal to the true initial robot pose.

Data files

Use the same data files as in challenge (A).

Report

Report the same comparisons as in challenge (A). Annotate if there are significant differences between the results obtained in challenges (A) and (B).