State Estimation and Filtering
Homework # 1

1. Simulate a sequence of independent two-dimensional random vectors e_i, $i = 1, \ldots, N$, with $N = 100$, such that e_i are Gaussian random variables with $E[e_i] = 0$ and

$$E[e_i e_i^T] = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}.$$

2. Simulate a sequence of one dimensional independent random variables x_i, uniformly distributed in the interval $[-1, 1]$ and independent from variables e_i. Then, define the sequence of two-dimensional random vectors y_i such that

$$\begin{aligned} y_{1,i} &= x_i + e_{1,i} \\ y_{2,i} &= 3x_i + e_{2,i} \end{aligned} \qquad \text{for } i = 1, \ldots, N. \tag{1}$$

3. By using equations (1) and the statistical properties of the involved random variables, compute the LMSE estimator of the random variable x based on a single measurement of the vector $y = [y_1 \; y_2]'$, and the corresponding MSE.

4. Now assume that the equations (1) and the statistical properties of the random variables are not known, but only the simulated sequences x_i and y_i are available. Compute the LMSE estimator by calculating the necessary sample means and covariances. Compare the resulting estimator with that obtained at point 3.

5. Repeat point 4 by simulating a data sequence of length $N = 10000$. What are the main differences? Explain.