Recursive parametric estimation

Linear regression model

$$y(t) = \varphi^T(t)\,\theta + e(t)$$

Recursive least squares estimate:

$$\hat{\theta}_t = \hat{\theta}_{t-1} + L(t)\left[y(t) - \varphi^T(t)\,\hat{\theta}_{t-1}\right]$$

where $L(t) = R^{-1}(t)\,\varphi(t)$

$$R(t) = R(t-1) + \varphi(t)\varphi^T(t)$$

Requires $O(d^3)$ operations due to inverse of matrix R.

Idea: propagate $P = R^{-1}$

$$R^{-1}(t) = R^{-1}(t-1) - R^{-1}(t-1)\varphi(t)\cdot$$
$$\cdot\left[1 + \varphi^T(t) R^{-1}(t-1)\varphi(t)\right]^{-1}\varphi^T(t) R^{-1}(t-1)$$

RLS algorithm (complexity $O(d^2)$)

$$\hat{\theta}_t = \hat{\theta}_{t-1} + L(t)\left[y(t) - \varphi^T(t)\hat{\theta}_{t-1}\right]$$

where

$$L(t) = P(t)\varphi(t) = \frac{P(t-1)\varphi(t)}{1+\varphi^T(t)P(t-1)\varphi(t)}$$

$$P(t) = P(t-1) - \frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{1+\varphi^T(t)P(t-1)\varphi(t)}$$

RLS is KF applied to

$$x(t+1) = x(t) = \theta$$
$$y(t) = \varphi^T(t)x(t) + v(t)$$

Extensions

→ slowly time-varying parameters
$\left(x(t+1) = x(t) + w(t)\right., \quad \left.E[w(t)w^T(t)] = Q\right)$

→ exponential data weighting

Exponential data weighting

Find θ that minimizes

$$\sum_{k=1}^{t} \lambda^{t-k}\left(y(k)-\varphi^T(k)\theta\right)^2 + \lambda^t \underbrace{(\theta-\hat{\theta}_0)^T P(0)^{-1}(\theta-\hat{\theta}_0)}_{\substack{\text{initial condition} \\ \hat{\theta}_0,\, P(0)}}$$

$\lambda \in (0,1]$: forgetting factor
(tipical values in the range
$0.95 \div 0.999$)

Exponentially Weighted RLS:

$$\hat{\theta}_t = \hat{\theta}_{t-1} + L(t)\left[y(t) - \varphi^T(t)\,\hat{\theta}_{t-1}\right]$$

$$L(t) = \frac{P(t-1)\,\varphi(t)}{\lambda + \varphi^T(t)\,P(t-1)\,\varphi(t)}$$

$$P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\,\varphi(t)\,\varphi^T(t)\,P(t-1)}{\lambda + \varphi^T(t)\,P(t-1)\,\varphi(t)}\right]$$

$\rightsquigarrow$ prevents $P(t)$ from vanishing

$\rightsquigarrow$ small λ increases error variance