Kalman filter in prediction form

$$\hat{x}(t+1|t) = \left(A - AK(t)C\right)\hat{x}(t|t-1) + $$
$$+ AK(t)\,y(t) + Bu(t)$$

$$P(t+1|t) = A\,P(t|t-1)A^T + GQG^T$$
$$- AP(t|t-1)C^T\left[CP(t|t-1)C^T + R\right]^{-1}CP(t|t-1)A^T$$

Disrete Riccati Equation (DRE)

$$\tilde{x}(t+1|t) \triangleq x(t+1) - \hat{x}(t+1|t)$$

$$\tilde{x}(t+1|t) = \left(A - AK(t)C\right)\tilde{x}(t|t-1)$$
$$- AK(t)\,v(t) + Gw(t)$$

Asymptotic properties of Kalman filter

Result

Let A, B, C, G, Q, R be constant matrices.
Let (A, C) be detectable and (A, H) be stabilizable, where $GQG^T = HH^T$.

Then,

1) $$\lim_{t \to +\infty} E\left[\tilde{x}(t+1|t)\right] = \emptyset$$

2) $$\lim_{t \to +\infty} P(t+1|t) = P_\infty < +\infty \quad \forall P(0|-1) \text{ positive definite}$$

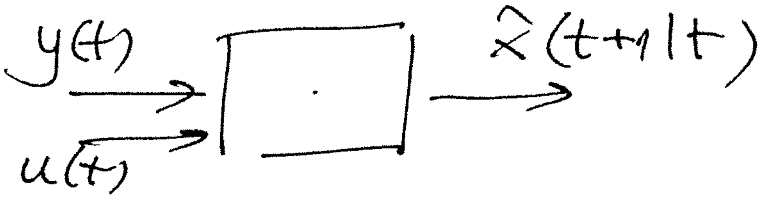
where P_∞ is the unique positive semidefinite solution of the Algebraic Riccati Equation (ARE)

$$P_\infty = AP_\infty A^T + GQG^T - AP_\infty C^T \left[CP_\infty C^T + R\right]^{-1} \cdot CP_\infty A^T$$

(same as DRE with $P(t+1|t) = P(t|t-1) = P_\infty$)

3) the matrix $(A - AK_\infty C)$ is asymptotically stable, with $K_\infty = \lim_{t \to +\infty} K(t)$

If the system is asymptotically stable (i.e, eigenvalues of A inside the unit circle), then $x(t)$ is asymptotically stationary and the LTI filter with $K(t) = K_\infty \ \forall t$ is the Wiener predictor:



$$\hat{x}(t+1|t) = (A - AK_\infty C)\,\hat{x}(t|t-1) + \begin{bmatrix} AK_\infty & B \end{bmatrix} \begin{bmatrix} y(t) \\ u(t) \end{bmatrix}$$

matlab → dlqe

(it gives P_∞ and K_∞ from A, C, G, Q, R)