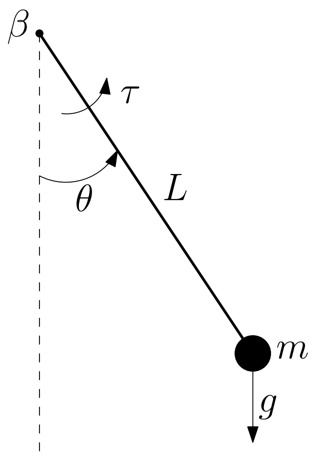
Simple Pendulum

Description and modeling

The system consists in a simple pendulum, as depicted below. A mass is connected to a rigid rod which is suspended from fixed point. Let θ be the angle of the rod with respect to the vertical axis, L the length of the rod and m the mass of the body. We consider that the mass of the rod is negligible. The friction is modeled as a linear dynamic friction with coefficient β acting on the pivot. Let the input to the system be the torque τ applied to the rod. So, the model can be written as:

$$mL^2\ddot{\theta} = -\beta\dot{\theta} - mgL\sin\theta + \tau$$

Numerical values: $m = 1,\ L = 2,\ \beta = 2,\ g = 9.8$



Experience Goal

- Control the pendulum angle in order to track a given reference signal.

Assignments

1. Build a Simulink model implementing the simple pendulum.

   Hint: provide an output related to the system states (it will be useful later).

   Hint: to derive a signal, use a *transfer function* block $D(s) = \dfrac{s}{s/1000 + 1}$.

2. Linearize (with standard linearization techniques) the system around the equilibrium point $\theta_0 = \pi$, $\dot{\theta}_0 = 0$.

3. Find the values for which a proportional controller stabilizes the linearized system.

4. Choose a proportional controller which stabilizes the system and simulate it both on the nonlinear model and on the linearized one. Comment the result.

5. Design a controller based on I/O Feedback linearization.

   Assume the following reference (warning: y_d is expressed in degrees)

   $$y_d(t) = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t + \phi)$$

   with $A_1 = 100,\ \omega_1 = \dfrac{2\pi}{10},\ A_2 = 70,\ \omega_2 = \dfrac{2\pi}{3},\ \phi = \dfrac{\pi}{2}$.

6. Find the intervals of the controller parameters which guarantee asymptotic stability of the controlled system.

7. Change the controller parameters and comment the results.

8. Change the reference signal (try steps, square waves, etc.) and comment.

9. Insert an additive noise (e.g., a white noise) on the states and evaluate the performance of the previously designed controller. Try with disturbances of different amplitude.