
1) \[ X_1(k) = y(k) \]
\[ X_2(k) = y(k+1) \]

\[ X_1(k+1) = y(k+1) = X_2(k) \]
\[ X_2(k+1) = y(k+2) = -\frac{1}{2}y(k+1) + \frac{1}{2}y(k) + u(k) \]

\[ y(k) = X_1(k) \]

\[ A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = [0] \]

2) \[ y(\emptyset) = -\frac{1}{2}y(-1) + \frac{1}{2}y(-2) + u(-2) = \frac{1}{2} \]

\[ y(1) = -\frac{1}{2}y(\emptyset) + \frac{1}{2}y(-1) + y(-1) = -\frac{1}{4} \]

\[ x(\emptyset) = \begin{bmatrix} y(\emptyset) \\ y(1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{4} \end{bmatrix} \]

\[ Y_z(z) = C (zI-A)^{-1} z.x(\emptyset) = \]
\[ = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z & -1 \\ -\frac{1}{2} & z+\frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{4} \end{bmatrix} = \]
\[ = \begin{bmatrix} 1 & 0 \end{bmatrix} \left( \frac{2+z}{2} \right)^{-1} \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{4} \end{bmatrix} = \]
\[ = \frac{2^2 + \frac{1}{2} \cdot \frac{1}{2}}{2} \cdot \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{4} \end{bmatrix} = \]
\[ = \frac{2^2 + \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2}}{2} \]
\[
\begin{align*}
\frac{\left[ z + \frac{1}{2} \ 1 \right]}{z^2 + \frac{1}{2} z - \frac{1}{2}} &= \frac{z \left( \frac{1}{2} z + \frac{1}{4} - \frac{1}{4} \right)}{z^2 + \frac{1}{2} z - \frac{1}{2}} \\
&= \frac{1}{2} \frac{z^2}{z^2 + \frac{1}{2} z - \frac{1}{2}}
\end{align*}
\]

\[
Y_0(z) = \frac{1}{2} \frac{z}{z^2 + \frac{1}{2} z - \frac{1}{2}} = \frac{R_1}{z - \frac{1}{2}} + \frac{R_2}{z + 1}
\]

\[
z^2 + \frac{1}{2} z - \frac{1}{2} = 0 \quad \Rightarrow \quad z = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2}}{2} = < \frac{1}{2}
\]

\[
\frac{R_1(z+1) + R_2(z - \frac{1}{2})}{(z - \frac{1}{2})(z+1)} = \frac{\frac{1}{2} z}{z^2 + \frac{1}{2} z - \frac{1}{2}}
\]

\[
R_1^2 + R_1 + R_2 z - \frac{1}{2} R_2 = \frac{1}{2} z \quad \Rightarrow \quad R_1 + R_2 = \frac{1}{2} \quad \Rightarrow \quad R_1 - \frac{1}{2} R_2 = 0
\]

\[
R_2 = \frac{1}{3} \quad R_1 = \frac{1}{6}
\]

\[
Y_0(z) = \frac{1}{6} \cdot \frac{2}{z - \frac{1}{2}} + \frac{1}{3} \cdot \frac{z}{z + 1}
\]

\[
Y_0(k) = \left\{ \frac{1}{6} \cdot \left( \frac{1}{2} \right)^k + \frac{1}{3} \cdot (-1)^k \right\} \delta(k)
\]

Approccio alternativo: fare direttamente la trasformata Zeta dell'equazione i/o, tenendo conto delle condizioni iniziali (per prescrizione!)
3) \[ G(z) = \frac{1}{z^2 + \frac{1}{2}z - \frac{1}{2}} \]

\[ Y_f(z) = G(z) \cdot \frac{z}{z-1} = \frac{1}{z^2 + \frac{1}{2}z - \frac{1}{2}} \cdot \frac{z}{z-1} \]

\[ \vdots \]

4) \[ u(k) = (-1)^k \cdot 1(k) \rightarrow U(z) = \frac{z}{z+1} \]

\[ Y_f(z) = G(z) \cdot U(z) = \frac{1}{z^2 + \frac{1}{2}z - \frac{1}{2}} \cdot \frac{z}{z+1} \]

\[ \left( z - \frac{1}{2} \right) \left( z + 1 \right) \]

\[ \frac{1}{2}, \mu = 1 \rightarrow \text{mod} \ (\frac{1}{2})^k \]

\[ -1, \mu = 2 \rightarrow \text{mod} \ (-1)^k, \ k \cdot (-1)^k \]

\[ \rightarrow y_f(k) \text{ diverge} \]
1) \( W(z) = \sum_{k=0}^{\infty} w(k) \cdot z^{-k} = \)

\[
1 \cdot z^{-0} - 1 \cdot z^{-1} + 3\sigma^2 z^{-2} - 3\sigma^2 z^{-3} + 9\sigma^4 z^{-4} - 8\sigma^4 z^{-5} + 2 + 6\sigma^6 z^{-6} - 2 + 6\sigma^6 z^{-7} + \ldots + \ldots
\]

\[
= \left\{ 1 + 3\sigma^2 z^{-2} + 9\sigma^4 z^{-4} + 2 + 6\sigma^6 z^{-6} + \ldots \right\} +
- 2^{-1} \left\{ 1 + 3\sigma^2 z^{-2} + 9\sigma^4 z^{-4} + 2 + 6\sigma^6 z^{-6} + \ldots \right\} =
\]

\[
= (1 - 2^{-1}) \left\{ \ldots \right\}
\]

\[
= (1 - 2^{-1}) \cdot \sum_{k=0}^{\infty} (\sqrt{3}\sigma z^{-2k})^2 =
\]

\[
= (1 - 2^{-1}) \cdot \sum_{k=0}^{\infty} (3\sigma^2 z^{-2k})^k =
\]

\[
= (1 - 2^{-1}) \cdot \frac{1}{1 - 3\sigma^2 z^{-2}} = \frac{z^2 - 2}{2^2 - 3\sigma^2}
\]

2) poli: \( z^2 - 3\sigma^2 = 0 \)  \( z = \pm \sqrt{3}\sigma \)  \((\sigma > 0)\)

mot: \( (\sqrt{3}\sigma)^k \) \( (-\sqrt{3}\sigma)^k \)

\(|\sqrt{3}\sigma| < 1 \Rightarrow 10^1 < \frac{1}{\sqrt{3}}\)